Pre-emptive sovereign debt restructuring and holdout litigation

Kartik Anand[a] and Prasanna Gai[b]

[a]*Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Germany, e-mail:* kartik.anand@bundesbank.de

[b]*University of Auckland, 12 Grafton Road, Auckland 1142, New Zealand, e-mail:* p.gai@auckland.ac.nz

Abstract

We offer an analytical framework for studying 'pre-emptive' debt exchanges. Countries can tailor a sovereign bankruptcy framework by choosing provisions (or 'haircuts') ex ante, but must contend with the market discipline of holdout litigation ex post. Secondary markets play a role in shaping the holdout costs facing the sovereign, and our results suggest that it is optimal to prioritise the rights of holdout creditors during litigation so that they are always paid in full. We clarify how macroeconomic and legal factors influence the choice of haircut. Our model contributes to the debate on sovereign debt restructuring by formalising Bolton and Skeel's notion of a 'Designer SDRM'.

Introduction

Debates on sovereign debt restructuring typically emphasise a trade-off between ex ante and ex post efficiency. Proponents of bankruptcy procedures (e.g. Krueger, 2002; IMF, 2002) highlight the ex post inefficiency posed by costly default, and emphasise the importance of mitigating these via institutional or market-based remedies. Critics of such proposals (e.g. Dooley, 2000; Shleifer, 2003) counter that sovereign debt is feasible and affordable only because of the threat of costly crises. Bankruptcy mechanisms that lower ex post costs of debt crises or shield the debtor from the threat of holdout litigation may do more harm than good by increasing moral hazard problems ex ante.

Statutory approaches to balancing ex ante incentives and ex post crisis costs have tended to highlight a 'double trigger' mechanism for a viable sovereign bankruptcy regime (Gai et al., 2004; Brookings-CIEPR, 2013).[1] A sovereign first makes a request for assistance, in a manner akin to filing for bankruptcy protection under Chapter 11 of the US bankruptcy code, and then a bankruptcy court (or the IMF) rules on the case. Gai et al. (2004) show formally that statutory mechanisms enhance welfare even though the bankruptcy court is only able to imperfectly judge if repayment problems are the result of genuine bad luck on the part of the sovereign.

In an important contribution to the debate, Bolton and Skeel (2004) question the feasibility of statutory approaches to sovereign debt restructuring. Drawing on US bankruptcy history, they suggest that tailored approaches may be more politically palatable. Allowing sovereigns to have the choice of opting out of the provisions of a bankruptcy framework depending on their economic circum-

[1] Other contributions that examine how policy interventions in sovereign lending influence both ex ante and ex post efficiency include Ghosal and Miller (2003) and Bolton and Jeanne (2007).

stances would increase their willingness to adopt a sovereign bankruptcy framework. Bolton and Skeel (2004) also emphasise the importance of preventing the dilution of the claims of existing creditors to ensure ex ante efficient levels of borrowing. The enforcement of absolute priority emerges as a key element of a robust sovereign bankruptcy framework.

The changing nature of holdout litigation has also altered the balance between ex ante and ex post efficiency. Ad hoc debt exchanges in the past were largely smooth because a sovereign could unilaterally bypass holdout creditors through take-it-or-leave-it offers backed by the agreement of a simple majority of bondholders. But the New York District Court rulings on Argentina in 2012 have made it easier for holdout creditors to litigate other creditors and, hence, indirectly harass the sovereign (Brookings-CIEPR, 2013). By lowering the costs of litigation, such developments have made participation in debt exchanges harder to coordinate.

In this paper, we present a model in which countries can tailor a sovereign bankruptcy framework to fit their own circumstances, but must contend with the market discipline of holdout litigation. Our model thus formalises elements of the Bolton and Skeel proposal, and contributes to the debate on whether sovereign debt restructuring mechanisms should be mandatory or permit opt-out options. This debate has become salient in recent years as the official sector has advanced the notion of 'pre-emptive' debt restructuring prior to default as a way of preventing countries from entering into post-default restructurings that are 'too little and too late' (IMF, 2013).

In our optimal contracting framework, a sovereign issues bonds to fund a risky project and chooses the bankruptcy provisions and exemptions (a 'haircut') ex ante to maximise domestic consumption. The ex ante choice of the haircut has two important incentive effects ex post. First, it influences the sovereign's willingness to repay. All other things being equal, a larger haircut means that the sovereign can more easily restructure its debts. This, in turn, increases the sovereign's incentives to default and file for bankruptcy. The second effect is on the creditors' incentives to holdout during the restructuring. In our model, creditors are ex post heterogeneous and have different costs of pursuing litigation against the sovereign. Litigants who reject the sovereign's haircut on their claims petition a bankruptcy judge that they be repaid in full. The judge's ruling reflects the extent to which creditors' rights are favored in the restructuring mechanism. Successful litigants are paid in full, while unsuccessful litigants walk away with nothing. Thus, a large haircut means that more creditors will holdout and pursue litigation. The sovereign seeks to mitigate the costs to output from default and holdouts and so offers a lower haircut in equilibrium.

The optimal haircut is shaped by creditors' ability to trade their claims in a secondary debt market prior to the restructuring phase. If the ex ante haircut is high, then creditors with high litigation costs have a strong incentive to sell their bonds to more litigious creditors, i.e., those with low litigation costs. This, in turn, exacerbates holdouts and litigation. In our model, if the most litigious creditor has deep pockets and sufficiently low marginal litigation costs, they can buy all the bonds from the other creditors. But if the ex ante haircut is low, then the incentives for high-litigation cost creditors to sell their claims to more litigious creditors are eliminated and there is no trade in the secondary market.

Our model predicts that countries with weak and risky macroeconomic fundamentals will voluntarily seek lower haircuts (i.e. harsher bankruptcy provisions) ex ante. Adverse shifts in the distribution of output have the effect of enlarging the range of macroeconomic outcomes over which the sovereign prefers to default. But if the threat of litigious holdout creditors is material, then the sovereign counters adverse holdout costs by offering a lower haircut ex ante.

The legal environment also influences the choice of haircut. If the bankruptcy regime is more creditor-friendly then two effects come into play that lower the ex ante haircut. First, an improvement in the rights of holdout creditors implies that it is more likely that the sovereign must repay holdout creditors. This reduces the incentive to file for bankruptcy. At the same time, creditors' incentives to holdout and litigate increase. To counter this, the sovereign lowers the ex ante haircut, offering more to creditors. To the extent that the ability of the bankruptcy framework to mitigate the output costs imposed by holdout creditors is increased, this raises the incentive for the sovereign to default and file for bankruptcy. This, in turn, enables the sovereign to increase the haircut and offer less to creditors during the restructuring.

Finally, our model suggests that it is optimal to prioritise the rights of holdout creditors during litigation so that they are always paid in full. This is consistent with the notion of strict adherence to absolute priority during debt restructuring that Bolton and Skeel also emphasise in their proposal.

Our work is related to the literature on creditor litigation in sovereign debt, notably Haldane et al. (2005), Engelen and Lambsdorff (2009), Pitchford and Wright (2012), Bai and Zhang (2012), and Schumacher et al. (2015). But these papers focus on the ex post renegotiation stage rather than the ex ante implications for sovereign borrowing and the design of the bankruptcy framework. Recent work has also highlighted the importance of the creditor base in sovereign debt enforcement. Broner et al. (2010) develop a model in which, once it becomes apparent that default is looming and penalties are insufficient, foreign creditors can sell debt securities on the secondary market to domestic residents. Since domestic residents expect the government to enforce domestic debts, they purchase the securities at face value. Trading in the secondary market thus allows foreign creditors to circumvent default. The secondary market channel through which debt is enforced in our model is very different, however.

Our paper is also related to recent work on pre-emptive sovereign debt restructuring. While many papers simply assume that restructurings are preceded by a default, Asonuma and Trebesch (2016) assume that this is not the case. They develop a quantitative sovereign debt model that emphasises the Nash bargaining game between the debtor and its creditors, and which incorporates the possibility of both pre-emptive and post-default restructuring. Asonuma and Trebesch show that pre-emptive restructuring is optimal if the sovereign expects a high probability of default and faces high output costs as a result. In their model, pre-emptive deals also lead to lower haircuts, quicker renegotiations, and lower crisis costs. Empirical studies of pre-emptive restructuring include Diaz-Cassou et al. (2008), Panizza et al. (2009), and Erce (2013).

Model

A small open economy extends over two periods, $t = 0, 1$. There is a single consumption good and a representative domestic agent, the sovereign, who is risk neutral and cares only about consuming in period 1.

At $t = 0$, the sovereign can borrow funds from the international capital market to finance a risky project that yields θ units of output at $t = 1$. We suppose that the sovereign issues a unit of debt under international law – an infinitely divisible one period bond that pays $R > 1$ on maturity at $t = 1$. The realised ex post return on the project is a random variable, $\theta \in [\underline{\theta}, \bar{\theta}]$, drawn from the cumulative distribution function $G(\theta)$. There are $n \in \mathbb{N}$ foreign creditors who are ex ante identical and risk neutral. These creditors are not capital constrained and are able to borrow and lend as much as needed at the constant risk-free world interest rate, $\bar{r}$.

In the presence of absolute sovereign immunity in international law, it is difficult (if not impossible) for foreign creditors to 'attach' a sovereign's property in the event of a default. Even though trading and credit relationships may be impaired in ways that diminish the sovereign's output as a result of default, lenders receive nothing. Denoting the fraction of output lost due to default by $\delta \in [0, 1]$, the sovereign is only willing to repay creditors in full provided

$$\theta - R > (1 - \delta)\theta\,. \tag{1}$$

To mitigate the costs associated with default, we allow the sovereign to tailor bankruptcy procedures ex ante. Specifically, the contract with foreign creditors at $t = 0$ stipulates a haircut, $h \in (0, 1)$, that creditors will be offered at $t = 1$ in the event that the sovereign files for bankruptcy in international courts to seek a restructuring. Filing for bankruptcy has two effects. First, it helps reduce the output loss from default to $\sigma\delta \leq \delta$, where $\sigma \in [0, 1]$ captures the ability of the bankruptcy mechanism to shield the sovereign from output losses. But this also has the effect of increasing the sovereign's incentive to default. Second, the recourse to a bankruptcy procedure ensures that creditors receive some payment in states of the world where they would otherwise have received nothing.

If the sovereign files for bankruptcy at $t = 1$, creditors are offered a haircut, h, on their claim. Each creditor must decide whether to accept the haircut or litigate in the courts to recover the full claim. We assume a per unit claim litigation cost, $\ell_i \in (0, 1)$ for each creditor i such that if creditor i's claim is R_i, then the cost of litigation is $R_i\ell_i$. The assumption that the cost of litigation scales with the size of the claim is consistent with economic analyses of litigation behaviour.[2] The marginal litigation costs, $\ell_1, \ell_2, \ldots, \ell_n$, are independently and identically drawn at the start of $t = 1$ from a uniform distribution for each creditor.

[2] Katz (1988) argues that litigating parties use more units of legal resources when the stakes are higher. See also Posner (1973) and Bebchuk (1984).

$t = 0$	$t = 1$
1. Sovereign issues bonds with repayment R and haircut h	1. Output and litigation costs realised 2. Sovereign repays or defaults 3. If sovereign repays: - Creditors consume R - Sovereign consumes $\theta - R$ 4. If sovereign defaults: - Creditors trade bonds at price P - Litigious creditors reject haircut h - Litigations succeed with probability ϕ - Non-holdouts consume $R(1-h)$ - Successful litigants consume R - Sovereign consumes remainder

Table 1: Timeline of events

Litigation is risky since the full claim can only be recovered with success probability ϕ. Litigation fails with probability $1-\phi$, in which case the holdout creditor receives nothing. Following Schumacher et al. (2015), ϕ can be interpreted as a measure of the strength of creditor rights. Holdouts are costly for the economy and reduce output by the fraction $f(\mu) \in [0, \eta]$, where μ is the ratio of holdout creditors to all bondholders. The cost of holdouts is increasing in μ, and $f(1) = \eta$.

Finally, creditors have access to a secondary market at $t = 1$. Upon discovering their litigation cost, creditors with high litigation costs may opt to sell their claims to those with low litigation costs. The secondary market for bonds is perfectly competitive with market-clearing price, P, per unit claim.

Table 1 illustrates the sequence of events in the model.

Equilibrium

We solve the model by backward induction. The pure strategy sub-game perfect Nash equilibrium consists of a bankruptcy threshold, θ^*, litigation threshold, ℓ^*, ratio of holdout creditors to all bondholders, μ^*, secondary market clearing price, P^*, repayment, R^*, and haircut, h^*, such that at $t = 1$

- conditional on the sovereign filing for bankruptcy, individual creditors litigate whenever $\ell_i < \ell^*$, given the repayment, R^* and haircut h^*;
- the secondary market clears so that aggregate demand and supply of bonds yield the equilibrium price P^*;
- creditors optimally choose to either demand or supply bonds, given their litigation costs, $\{\ell_i\}_{i=0}^1$, the price, P^*, the repayment, R^*, and the haircut h^*;
- the sovereign files for bankruptcy if $\theta < \theta^*$, given the mass of holdout creditors, μ^*, repayment, R^*, and haircut h^*;

And, at $t = 0$,

- the sovereign chooses the haircut, h^*, to maximise expected consumption;
- creditors determine the repayment, R^*, from their participation constraint.

Incentives to holdout

If the sovereign files for bankruptcy and offers creditors a haircut, h, then an individual creditor, i, who accepts the offer receives $R_i(1 - h)$. But if the creditor rejects the offer and litigates with success probability ϕ, the expected payoff after incurring litigation costs is $R_i(\phi - \ell_i)$. So creditor i holds out whenever

$$R_i(\phi - \ell_i) > R_i(1 - h)\,, \tag{2}$$

i.e., whenever $\ell_i < \ell^* \equiv h - (1 - \phi)$.

Secondary market for bonds

Upon learning their litigation costs at $t = 1$, creditors can buy or sell bonds in a perfectly competitive secondary market. Since all creditors are ex ante identical, they each hold an equal share, R/n, of the bond. Without loss of generality, we can order all creditors based on their marginal litigation costs. The first creditor has the smallest litigation cost, ℓ_1, while the second creditor has the second smallest litigation cost, ℓ_2, and so on. We thus have, $\ell_1 < \ell_2 < \ldots < \ell_n$. Suppose that the first m creditors all have a marginal litigation cost less than ℓ^*. It would be optimal for each of these creditors to reject the offered haircut, h, and litigate against the sovereign. While, for the remaining $n-m$ creditors, it would be optimal to accept the haircut. But, with a secondary market, these $n - m$ creditors prefer to sell their claims to the m litigious creditors instead. In equilibrium, insofar as $\ell_1 < \ell^*$, all other creditors sell their claims to the first creditor.

Proposition 1. *If the first creditor has a small enough marginal litigation cost, $\ell_1 < \ell^*$, then bonds are traded in the secondary market. The first creditor buys all bonds, and the equilibrium bond price is $P^* \in \left(\frac{R}{n}(\phi - \ell_2), \frac{R}{n}(\phi - \ell_1)\right]$. If, however, the marginal litigation cost for the first creditor is large, $\ell_1 \geq \ell^*$, then it is optimal for all creditors to accept the haircut, and there is no trade.*

Proof. See Appendix. □

The equilibrium bond price is indeterminate. However, since all creditors are risk-neutral, the exact price is immaterial when deriving the participation constraint. What matters, instead, is whether or not the first trader is litigious, $\ell_1 < \ell^*$, thus satisfying the condition for trade.

Sovereign's willingness to pay

Turning to the sovereign's decision to file for bankruptcy, there are two cases to consider: (i) when bonds are traded, i.e $\ell_1 < \ell^*$; and (ii) when bonds are not traded, i.e $\ell_1 \geq \ell^*$. We consider each in turn.

If the first creditor has a small enough litigation cost, then bonds are traded in the secondary market. The first creditor buys all bonds from all other creditors. The first creditor subsequently rejects the haircut and pursues litigation, so $\mu^* = 1$. In this case, the willingness to pay constraint is

$$\theta - R > \big(1 - \sigma\delta - \eta\big)\theta - R\phi\,. \tag{3}$$

This can be arranged to obtain that the sovereign will repay whenever the output, is greater than a bankruptcy threshold, i.e., $\theta > \theta^{**} \equiv \frac{R(1-\phi)}{\sigma\delta+\eta}$.

If the first creditor's marginal litigation cost is greater than ℓ^*, then it is optimal for all creditors to accept the haircut, h. As such, there are no holdout creditors, and the sovereign's willingness to pay constraint is

$$\theta - R > \big(1 - \sigma\delta\big)\theta - R(1-h)\,. \tag{4}$$

On rearranging, this yields that the sovereign will repay in full whenever output is greater than an alternate bankruptcy threshold, i.e., $\theta > \theta^*(h) \equiv \frac{Rh}{\sigma\delta}$. We can order the two thresholds such that $\theta^{**} < \theta^*$.

Proposition 2 summarises the equilibrium at $t = 1$.

Proposition 2. *The equilibrium ratio of holdout creditors to all bondholders is*

$$\mu^* = \begin{cases} 1 & \text{if} \quad \ell_1 < \ell^*(h) \\ 0 & \text{otherwise} \end{cases}.$$

The secondary market bond price is

$$P^* = \begin{cases} \left[\frac{R}{n}(\phi - \ell_2), \frac{R}{n}(\phi - \ell_1)\right] & \text{if} \quad \ell_1 < \ell^*(h) \\ \text{No trade} & \text{otherwise} \end{cases}.$$

And the resulting sovereign bankruptcy threshold is

$$\begin{cases} \theta^{**} & \text{if} \quad \ell_1 < \ell^*(h) \\ \theta^*(h) & \text{otherwise} \end{cases},$$

where $\theta_h^* > 0$.

Insofar as $\ell_1 < \ell^*(h)$, a marginal increase in the haircut has no further effect on the equilibrium. But if $\ell_1 \geq \ell^*(h)$ then an increase in the haircut has potentially two effects. First, the bankruptcy threshold, $\theta^*(h)$, increases, implying that the sovereign is more likely to file for bankruptcy even for larger realizations of the output. At the same time, the critical litigation threshold, $\ell^*(h)$, increases. As long as ℓ_1 remains above the threshold, the secondary market remains closed, and all creditors choose to accept the haircut. But once ℓ_1 falls below the threshold the equilibrium switches to one where there is an active secondary market; the first creditor buys all bonds from all other creditors and the first creditor rejects the haircut and litigates.

Optimal haircut

At $t = 0$, the sovereign chooses the haircut to maximise expected consumption, which is expected output net of bankruptcy and holdout costs, and repayments to creditors. Denoting the unconditional expected output by $\widehat{\theta}$, expected consumption, EC, is given by

$$\begin{aligned} EC(h) &= F_1(\ell^*(h))\left\{\widehat{\theta}-\big(\sigma\delta+\eta\big)\int_{\underline{\theta}}^{\theta^{**}}\theta dG(\theta)-R\big(1-G(\theta^{**})(1-\phi)\big)\right\} \\ &+ \big[1-F_1(\ell^*(h))\big]\left\{\widehat{\theta}-\sigma\delta\int_{\underline{\theta}}^{\theta^*(h)}\theta dG(\theta)-R\big(1-G(\theta^*(h))h\big)\right\}, \quad (5) \end{aligned}$$

where

$$F_1(\ell)\equiv 1-\big(1-\ell\big)^n$$

is the cumulative distribution function for the smallest marginal litigation cost. If the sovereign expects that $\ell_1 < \ell^*(h)$ then, upon default, the first creditor will buy all bonds from all other creditors, reject the haircut and litigate. While, if the sovereign expects $\ell_1 \geq \ell^*(h)$, then all creditors accept the haircut, and there is no further output loss from holdouts. As Figure 1 illustrates, the expected consumption is a concave function in the haircut.[3] The optimal haircut, h^*, where expected consumption is maximised is depicted by the grey dotted line.

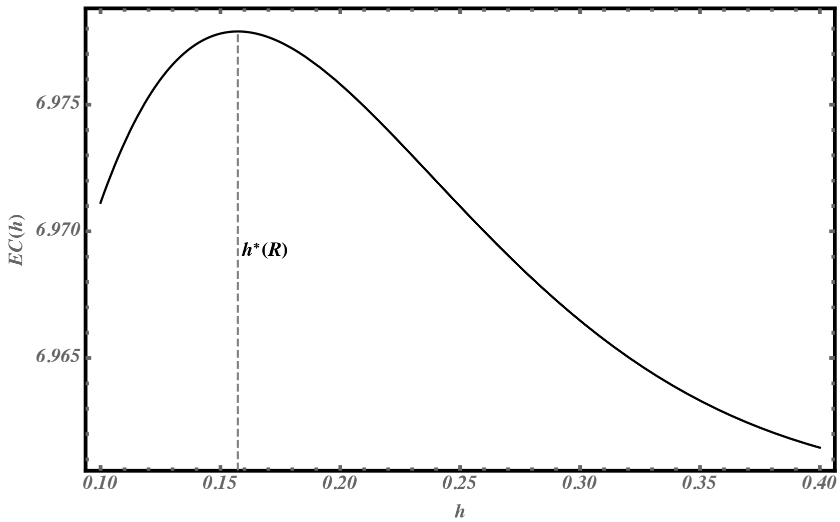

Figure 1: Expected consumption as a function of the haircut

[3]In producing the figure, we set the repayment at $R = 3$. For the output losses stemming from default and holdouts, we assume $\delta = 1$, $\sigma = 0.1$ and $\eta = 0.11$, respectively. The bankruptcy regime is creditor-friendly with $\phi = 0.9$. The number of foreign creditors is set at $n = 15$. Finally, output is uniformly distributed over the interval $[0.1, 20]$.

In what follows, we make the following three assumptions. First, output is uniformly distributed over $[\bar{\theta}, \underline{\theta}]$. Second, the bankruptcy regime is creditor friendly, $\phi > \tilde{\phi}$. And third, for a given $\bar{\theta}$, the expected output cannot be too large, implying $\underline{\theta} < \underset{\sim}{\theta}$. The first assumption is for analytical tractability. The second assumption ensures that the optimal haircut is an interior solution, while the third ensures that expected consumption is maximised at the optimal haircut. Proposition 3 summarises our results.

Proposition 3. *For any given repayment, there exists a unique haircut, $h^*(R)$, that maximises expected consumption. The optimal haircut is decreasing in the repayment. A leftward shift in the output distribution, i.e a decrease in $\underline{\theta}$, lowers the optimal haircut, as does increased riskiness (in the sense of a mean-preserving shift in the output distribution). The optimal haircut is lowered if the bankruptcy regime is more creditor-friendly, and if the maximal output costs that holdout creditors can inflict increases.*

Proof. See Appendix. □

As repayments increase, they induce two opposing effects on the sovereign's choice of haircut. First, an increase in R heightens the sovereign's incentive to file for bankruptcy. Second, insofar as the first creditor's marginal litigation cost is low, following the declaration of bankruptcy by the sovereign, all other creditors will sell their bonds to the first creditor who, in turn, rejects the haircut and pursues litigation. The first effect induces the sovereign to opt for a higher haircut, while the second effect acts in the opposite direction. From the first-order condition, the second effect always dominates.

Our model predicts that countries with weak and risky macroeconomic fundamentals will seek lower haircuts ex ante.[4] A leftward shift in the output dis-

[4]Our theoretical finding is consistent with the empirical evidence presented in Asonuma and

tribution lowers expected output. The range of θ over which the sovereign would default is larger. But, and as before, if the first creditor's marginal litigation cost is sufficiently small, the first creditor buys all bonds on the secondary market and litigates. This inflicts a large holdout cost for the sovereign. The sovereign can mitigate these costs by lowering the haircut, and thereby reducing the likelihood that the first creditor will be litigious.

Increasing the variance of the output distribution, while leaving the mean unchanged also has the effect of enlarging the range of θ over which the sovereign defaults and incurs holdout costs. The greater risk of suffering an output cost again induces a lower haircut.

An improvement in the creditor-friendliness of the bankruptcy regime induces two effects. First, it becomes more likely that the sovereign must repay holdouts. This reduces the sovereign's incentives to file for bankruptcy. And second, the creditors' incentives to holdout and litigate increase. To counter this, the sovereign lower the ex ante haircut, thereby offering more to creditors during the restructuring so as to offset their incentives to holdout.

Finally, an increase in maximal holdout cost reduces expected consumption whenever bonds are traded on the secondary market. The sovereign can mitigate these costs by reducing the likelihood that the first creditor's marginal litigation cost is below the threshold, $\ell^*(h)$. This, in turn, is achieved by reducing the haircut and offering more to creditors during the restructuring.

Pricing sovereign debt

The total repayment, R, due at $t = 1$ is determined by aggregating the break-even conditions of individual creditors. Since all creditors are ex ante identical,

Trebesch (2016). They find that countries with high default risk are significantly more likely to pre-emptively restructure their debts and agree to lower haircuts.

the 'representative' creditor holds a claim, $\frac{R}{n}$, against the sovereign. If the first creditor's marginal litigation cost is greater than the threshold, $\ell^*(h)$, then the representative creditor accepts the haircut during the restructuring if the sovereign files for bankruptcy. Integrating over output, the claim is worth

$$\frac{R}{n}\big(1 - G(\theta^*(h))\big) + \frac{R}{n}(1-h)G(\theta^*(h))\,.$$

Next, suppose that the first creditor's litigation cost is below $\ell^*(h)$. If the sovereign files for bankruptcy, there are two further cases to consider. First, if the representative creditor does not have the smallest marginal litigation cost (with probability $\frac{n-1}{n}$), then the creditor will sell its bonds for price P^*. Second, if the representative creditor has the smallest marginal litigation cost (with probability $\frac{1}{n}$), then the creditor will purchase all bonds from the other creditors for price P^*, per claim. The representative creditor will then reject the haircut and pursue litigation. Integrating over output, the value of holding a claim for the representative creditor is $\frac{R}{n}\big(1-G(\theta^{**})\big)+\left[\frac{1}{n}\left\{R(\phi-\ell_1)-(n-1)P^*\right\}+\frac{n-1}{n}P^*\right]G(\theta^{**})$ which, on rearranging, yields

$$\frac{R}{n}\big(1 - G(\theta^{**})\big) + \frac{R}{n}(\phi-\ell_1)G(\theta^{**})$$

Integrating over the first-creditor's marginal litigation cost, and aggregating over all creditors, the repayment, $R^* \equiv R^*(h)$, is given by the solution to

$$\begin{aligned} V(h,R^*) \;&\equiv\; R\Big\{1 - \big(1 - F_1(\ell^*(h))\big)G(\theta^*(h))h \\ &-\; F_1(\ell^*(h))G(\theta^{**})\Big[(1-\phi) + \int_0^{\ell^*(h)} \ell_d F_1(\ell)\Big]\Big\} - \big(1+\bar{r}\big) = 0\,. \end{aligned}$$

For $R^*(h)$ to be unique, we require that the world interest rate is sufficiently small, $\bar{r} < \underset{\sim}{r}$. Proposition 4 summarises the properties of $R^*(h)$.

Proposition 4. *For any given haircut, the repayment, $R^*(h)$ is unique. The repayment is increasing in the haircut. A decrease in expected output increases repayment. Greater creditor-friendliness of the bankruptcy regime decreases repayment. Finally, higher world interest rates also lead to greater repayment.*

Proof. See Appendix. □

An increase in the haircut has several effects. First, the likelihood that the first creditor will be litigious increases. It is more likely that creditors trade bonds on the secondary market if the sovereign files for bankruptcy, and for the first creditor to pursue litigation. At the same time, the expected litigation cost increases. Second, conditional on there being no secondary market trade, an increase in the haircut increases the sovereign's incentives to file for bankruptcy and settle with the creditors. To the extent that the secondary market effect is dominated and all creditors must accept the haircut, a higher repayment is required for creditors to break even.

The intuition behind the remaining results of Proposition 4 are more easily understood. A change to the output distribution that increases the risk of default (a fall in $\bar{\theta}$) leads to an increase in repayment demanded by creditors faced with greater default risk. As the creditor friendliness of the bankruptcy regime increases, the sovereign's incentives to default are reduced. Moreover, if the sovereign did file for bankruptcy, more creditors would holdout and claim the full repayment with greater probability. Thus, the value of creditors' claims against the sovereign increase and, hence, the required repayment decreases. Higher world interest rates increase the outside option for creditors who must, therefore, be compensated for sovereign lending with higher repayments.

We close the discussion of our model with a description of the joint equilibrium. The comparative statics directly follow from our earlier analysis of the schedules

$h^*(R)$ and $R^*(h)$ in Propositions 3 and 4, respectively.

Proposition 5. *The optimal haircut and repayment, i.e., h^{**} and R^{**}, respectively, jointly solves*

$$\begin{aligned} EC_h(h^{**}, R^{**}) &= 0 \\ V(h^{**}, R^{**}) &= 0\,. \end{aligned}$$

An increase in the world interest rate, $\bar{r}$, leads to a decrease in the haircut and an increase in the repayment. An increase in the maximum output, $\bar{\theta}$, leads to an increase in the haircut and a decrease in the repayment. An improvement in the creditor-friendliness of the bankruptcy regime leads to an increase in the haircut, but has an ambiguous effect on repayment. Finally, an increase in the maximal holdout cost has an ambiguous effect on h^{**} but leads to a decrease in R^{**}.

Implications for sovereign debt restructuring

Early proposals for sovereign bankruptcy frameworks (Krueger, 2002; IMF, 2002) typically assumed a 'one-size-fits-all' approach based on a uniform and mandatory set of provisions. In an important contribution to the debate, Bolton and Skeel (2004) propose that sovereigns be permitted to opt out of some aspects of the bankruptcy procedure ex ante. They argue that a tailored approach to sovereign bankruptcy is easier to sell politically than more ambitious statutory approaches. Debtor nations that typically express strong reservations about sovereign debt restructuring regimes may be more willing to participate if they are able to obtain exemptions to provisions in accordance with local conditions.[5]

[5] Bolton and Skeel (2004) base their argument on US bankruptcy history – to pass the first US bankruptcy law at the Federal level in the 19th century, each state could determine what collateral a debtor could exempt from its creditors if it filed for bankruptcy. State lawmakers could thus adjust their exemptions to local norms and had discretion over how much collateral a debtor could retain. Bankruptcy law was Federal, but tailored on a state-by-state basis.

But there is a risk that countries may adopt overly harsh provisions ex ante, for fear of losing their access to credit.

Our model helps shed light on this debate and the disciplining role played by holdout litigation in determining ex ante exemptions (or haircuts). Our findings suggest that tailored frameworks will vary across countries according to their default risk – countries with weak or risky macroeconomic fundamentals will be more likely to settle for harsher provisions. Debtors are also more likely to adopt harsh provisions ex ante, the more creditor-friendly is the bankruptcy regime and the greater the output losses that can be inflicted by holdout creditors. But it should be noted that, in our framework, contractual flexibility results in an efficient outcome. As such, we are unable to address the normative issue of whether sovereigns should be precluded from adopting overly harsh provisions that make restructuring more, rather than less, painful.[6]

The analysis also clarifies the extent to which a sovereign debt restructuring should favour creditors. To the extent that an international body seeks to maximise welfare, i.e. the sum of domestic expected consumption and repayment to creditor, it can choose the degree of creditor protection, ϕ, to maximise

$$\begin{aligned} W(R,h;\phi) &= \widehat{\theta} - F_1(\ell^*(h))\big(\sigma\delta+\eta\big)\int_{\underline{\theta}}^{\theta^{**}}\theta dG(\theta) - RF_1(\ell^*(h))G(\theta^{**})\int_0^{\ell^*(h)}\ell dF_1(\ell) \\ &- \big[1-F_1(\ell^*(h))\big]\sigma\delta\int_{\underline{\theta}}^{\theta^*(h)}\theta dG(\theta)\,, \end{aligned} \tag{6}$$

which is the sum of expected output, loss to the sovereign from default and holdouts, and the cost to creditors from litigation. The effect of a change in ϕ can be decomposed into direct and indirect effects. We treat each in turn.

[6] Myopic policymakers who quickly want to access international capital markets, but who are unlikely to be around to face the costs of future defaults, are likely to be more willing to advocate quick and creditor-friendly restructuring. Extending the model to capture such political distortion is one way to explore this issue. See Acharya and Rajan (2013) for a model in this vein.

First, for given h, R, $\theta^*(h)$, and θ^{**}, a marginal increase in ϕ implies that creditors who reject the sovereign's haircut offer in bankruptcy and litigate are more likely to succeed. This, in turn, implies that if the sovereign were to default, it is more likely that all other creditors will sell their bonds to the most litigious creditor, who will holdout. So the expected holdout cost to the sovereign and expected litigation cost to the litigant both increase, which reduces welfare.

The second effect of an increase in ϕ is on the haircut, repayment, and the sovereign's default incentives. As Proposition 5 indicates, following an increase in ϕ, the repayment decreases. Since the litigation cost scales with the size of the claim, this indirect effect via R helps improve welfare. At the same time, insofar as creditors sell their claims to the litigious creditor via the secondary market, the sovereign's incentives to default and file for bankruptcy are reduced. This helps lower the expected holdout costs, which also improves welfare. Finally, the effects via the optimal haircut are in general ambiguous. If the haircut also decreased, then this reduces the incentives of creditors to trade bonds and litigate against the sovereign in bankruptcy. This also improves welfare. However, if the haircut increases following a marginal increase in ϕ, then not only are the sovereign's incentives to default, $\theta^*(h)$, also increased, but so also are the incentives for creditors to trade bonds and litigate. This would reduce welfare.

To assess on how welfare depends on ϕ, we resort to numerical methods. Figure 2 plots the welfare as a function of ϕ.[7] The different curves depict different values of σ, which captures the ability of the bankruptcy mechanism to shield the sovereign from output losses. In particular, the lower is σ, the lower is the output loss to the sovereign from defaulting. We readily note that, irrespective of σ, welfare is always maximised at $\phi^* = 1$, i.e., whenever creditors litigate, they always succeed. This

[7] In producing the figure, we normalised the world interest rate to zero. For the output loss and holdout cost to the sovereign, we set $\delta = 1$ and $\eta = 0.11$. The number of foreign creditors is $n = 15$, and the output is uniformly distributed over $[0.1, 20]$.

suggests that the indirect effects via the haircut, repayment, and the sovereign's bankruptcy incentives dominate over the creditors' incentives to litigate and hold out. Furthermore, the maximal value of welfare is independent of σ.

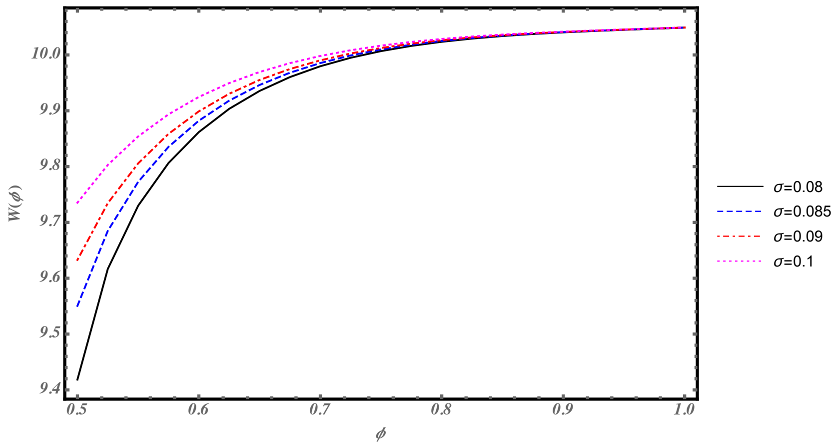

Figure 2: Welfare as a function of creditor friendliness

The situation $\phi^* = 1$ is consistent with the notion of absolute priority, namely the prioritisation by the bankruptcy regime of the payments of bonds issued first before those issued later. Bolton and Skeel (2004) emphasise the importance of enforcing absolute priority as the basis of any bankruptcy procedure and our model confirms this perspective. The lack of adherence to absolute priority is damaging for global welfare in our model since, by diluting the claims of existing creditors, it curbs market discipline and results in higher borrowing costs for sovereigns. Reinforcing creditor priorities within the bankruptcy mechanism, thus, helps balance the trade-off between ex post crisis costs and ex ante incentives.

Conclusion

Institutional solutions to sovereign debt restructuring entailing uniform and mandatory bankruptcy provisions are often criticised because there are too many political obstacles to implementation. As a result, contractual and market solutions have tended to be favoured by many policymakers and commentators.

In this paper, we have sought to examine a middle ground in which sovereigns are given the choice of opting out of a statutory bankruptcy framework by crafting their own provisions. We present a model that sheds light on the role played by holdout creditors, macroeconomic fundamentals, and the legal environment on the choice of exemptions (or haircut) sought ex ante. Our model formalises some of the insights of the Bolton and Skeel (2004) proposal advocating 'Designer SDRMs' as a possible transitory step on the road to a statutory bankruptcy regime.

The lack of sufficient support for a statutory mechanism has led the IMF to follow a related path, namely encouraging countries to restructure their debts pre-emptively (i.e. launched prior to default) in an effort to ensure orderly sovereign bankruptcy (IMF, 2013). Our paper thus contributes to a broader policy debate on mechanisms to ensure that unsustainable debt situations are recognised in a timely way. Extending the model to allow for government myopia would also allow analysis of whether sovereigns should be precluded from adopting overly harsh provisions.

Finally, the legal environment in our model is rudimentary. Enriching the analysis of sovereign debt enforcement to allow for judicial micro-foundations and a deeper understanding of sovereign immunity (in the spirit of Miller and Thomas, 2007 and Weidemaier and Gulati, 2015) is an important area for future work.

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Appendix

Proof of Proposition 1

Denote by $q_n^S(P)$ the quantity of bonds supplied by the n^{th} creditor at price P. Since the creditor does not litigate, the private value of the claim is $\frac{R}{n}(1-h)$. If the price satisfies $P > \frac{R}{n}(1-h)$, the creditor sells all $1/n$ claims. But, if $P < \frac{R}{n}(1-h)$, the creditor would never sell any bonds. Finally, if $P = \frac{R}{n}(1-h)$, the quantity of bonds supplied is indeterminate. Equation (.1) summarises.

$$q_n^S(P) = \begin{cases} 0 & \text{if} \quad P < \frac{R}{n}(1-h) \\ \left[0, \frac{1}{n}\right] & \text{if} \quad P = \frac{R}{n}(1-h) \\ \frac{1}{n} & \text{if} \quad P > \frac{R}{n}(1-h) \end{cases} . \tag{.1}$$

Likewise, let $q_n^D(P)$ denote the quantity of bonds demanded by creditor n at price P. This is given by

$$q_n^D(P) = \begin{cases} \frac{n-1}{n} & \text{if} \quad P < \frac{R}{n}(1-h) \\ \left[0, \frac{n-1}{n}\right] & \text{if} \quad P = \frac{R}{n}(1-h) \\ 0 & \text{if} \quad P > \frac{R}{n}(1-h) \end{cases} . \tag{.2}$$

As long as the price is less than the value of accepting the haircut, then creditor n would seek to buy all outstanding bonds. Since all non-litigious creditors have the same behaviour, and base their decisions on the size of the haircut, it follows that $q_{n-m}^S(P) = q_{n-m+1}^S(P) = \ldots = q_n^S(P)$, and $q_{n-m}^D(P) = q_{n-m+1}^D(P) = \ldots = q_n^D(P)$.

For the $n - m - 1^{\text{th}}$ creditor, with marginal litigation cost $\ell_{n-m-1} < \ell^*$, the supply and demand schedules are given by

$$q^S_{n-m-1}(P) = \begin{cases} 0 & \text{if} \quad P < \frac{R}{n}(\phi - \ell_{n-m-1}) \\ \left[0, \frac{1}{n}\right] & \text{if} \quad P = \frac{R}{n}(\phi - \ell_{n-m-1}) \\ \frac{1}{n} & \text{if} \quad P > \frac{R}{n}(\phi - \ell_{n-m-1}) \end{cases}, \tag{.3}$$

and

$$q^D_{n-m-1}(P) = \begin{cases} \frac{n-1}{n} & \text{if} \quad P < \frac{R}{n}(\phi - \ell_{n-m-1}) \\ \left[0, \frac{n-1}{n}\right] & \text{if} \quad P = \frac{R}{n}(\phi - \ell_{n-m-1}) \\ 0 & \text{if} \quad P > \frac{R}{n}(\phi - \ell_{n-m-1}) \end{cases}, \tag{.4}$$

respectively. As long as the price, P, is greater than $\frac{R}{n}(\phi - \ell_{n-m-1}) > \frac{R}{n}(1-h)$, the creditor is willing to buy all available bonds and sells none to other creditors. The supply and demand schedules for creditors $1, 2, \ldots,$ and, $n-m-2$ are similar to those described in Equations (.3) and (.4). Denoting the aggregate supply and demand schedules by $Q^S(P)$ and $Q^D(P)$, respectively, these are given by

$$Q^S(P) = \sum_{i=1}^{n} q^S_i(P), \qquad Q^D(P) = \min\left\{\sum_{i=1}^{n} q^D_i(P), \frac{n-1}{n}\right\}.$$

Proof for Proposition 3

The proof is in three steps. We first establish that there exists an optimal haircut, $h^* \in (1-\phi, 1)$, that solves $EC_h(h^*) = 0$. Second, we show that h^* is a maximum. Finally, we derive comparative statics on h^*.

The optimal haircut is given by the solution to $EC_h(h^*) = 0$, where

$$\begin{aligned} EC_h(h) \equiv\ & R\big(\theta^*(h) - \underline{\theta}\big) - \frac{n}{1-\ell^*(h)}\left[\frac{\sigma\delta + \eta}{2}\left((\theta^{**})^2 - \underline{\theta}^2\right)\right. \\ & - \left.\frac{\sigma\delta}{2}\left(\theta^*(h)^2 - \underline{\theta}^2\right) + R\Big(\big(\theta^*(h) - \underline{\theta}\big)h - \big(\theta^{**} - \underline{\theta}\big)(1-\phi)\Big)\right], \end{aligned} \tag{.5}$$

If the haircut is less than $1-\phi$, then it is never optimal for any creditor to litigate. Therefore, the sovereign can marginally increase the haircut without incurring any further cost. To show that the optimal haircut is greater than $1-\phi$, we require that $EW_h(1-\phi) > 0$. In particular,

$$\begin{aligned} EW_h(1-\phi) &= R\Big(\theta^*(1-\phi)-\underline{\theta}\Big) - n\Bigg[\frac{\sigma\delta}{2}\Big((\theta^{**})^2-\theta^*(1-\phi)^2\Big) \\ &+ \frac{\eta}{2}\Big((\theta^{**})^2-\underline{\theta}^2\Big) + R(1-\phi)\Big(\theta^*(1-\phi)-\theta^{**}\Big)\Bigg] \\ &= R\Big(\theta^*(1-\phi)-\underline{\theta}\Big) - n\Bigg[\frac{\eta}{2}\Big((\theta^{**})^2-\underline{\theta}^2\Big) \\ &+ \Big(\theta^*(1-\phi)-(\theta^{**})\Big)\Big\{R(1-\phi)-\frac{\sigma\delta}{2}\Big(\theta^*(1-\phi)+\theta^{**}\Big)\Big\}\Bigg]. \end{aligned}$$

It follows that

$$EW_h(1-\phi) > R\Big(\theta^*(1-\phi)-\underline{\theta}\Big)\left\{1-\frac{n\eta}{2}\left[\theta^{**}+\underline{\theta}+\frac{1-\phi}{\sigma\delta+\eta}\right]\right\}.$$

A strict sufficient condition for $EW_h(1-\phi) > 0$ is given by

$$\phi > \tilde{\phi} \equiv 1-\left(\frac{2}{n\eta}-\underline{\theta}\right)\frac{\sigma\delta+\eta}{\bar{\theta}+1}. \tag{.6}$$

Next, we need to ensure that $h^* < 1$, which we obtain if the number of creditors is large, i.e., $n \gg 1$. In this case, the probability of finding a creditor with a low litigation cost is high. Thus, if the haircut is large enough, expected consumption will be strictly decreasing for any further increase in the haircut.

To show that $h^*(R)$ is a maximum, we require that $EC_{hh}(h^*) < 0$. To this end, note that

$$EC_{hh}(h^*) = R\frac{\partial\theta^*}{\partial h} - \frac{R(n+1)}{1-\ell^*(h)}\big(\theta^*(h)-\underline{\theta}\big),$$

which is negative as long as $\underline{\theta} < \underset{\sim}{\theta} \equiv \frac{1-\phi}{\sigma\delta} - \frac{1}{\sigma\delta}$.

Finally, we evaluate how the optimal haircut varies with respect to changes in the repayment, R, output distribution, $\underline{\theta}$, creditor-friendliness, ϕ and holdout cost, η. To derive the effects, we need to first sign the cross-derivative terms:

$$\begin{aligned} EW_{hR}(h^*) &= \left(\frac{1-\ell^*(h^*)}{n} - \ell^*(h^*)\right) - \frac{\eta}{R}\underline{\theta}^2 < 0 \\ EW_{h\underline{\theta}} &= \eta\underline{\theta} - R\left(\frac{1-\ell^*(h^*)}{n} - \ell^*(h^*)\right) > 0 \\ EW_{h\phi} &\propto \underline{\theta} - R\left[\frac{h}{\sigma\delta} + \frac{n(1-\phi)}{\sigma\delta+\eta}\right] < 0 \\ EW_{h\eta} &= \frac{1}{2}\big(- (\theta^{*}{**})^2 + \underline{\theta}^2\big) < 0\,. \end{aligned}$$

It follows from the implicit function theorem that

$$\frac{\partial h^*}{\partial R} < 0\,, \quad \frac{\partial h^*}{\partial \underline{\theta}} > 0\,, \quad \frac{\partial h^*}{\partial \phi} < 0\,, \quad \text{and} \quad \frac{\partial h^*}{\partial \eta} < 0\,.$$

Proof of Proposition 4

The proof is in three steps. First, we establish that a solution, $R^*(h)$, that solves $V(h, R^*(h)) = 0$ exists. Second, we provide conditions under which this solution is unique. Finally, we derive the comparative statics.

Existence. Define $T(R) = \frac{1+\bar{r}}{A(R)}$ as a mapping from the set of possible repayments, $\mathcal{U}$, on to itself, where $A(R) \equiv \Big\{1 - \big(1 - F_1(\ell^*(h))\big)G(\theta^*(h))h - F_1(\ell^*(h))G(\theta^{**})\Big[(1-\phi) + \int_0^{\ell^*(h)} \ell_d F_1(\ell)\Big]\Big\}$. Since $\mathcal{U}$ is closed and compact, it follows from Brouwer's fixed point theorem that there exists at least one fixed-point for the mapping.

Uniqueness. We must establish three criteria to guarantee uniqueness of the equilibrium. First, that if the repayment is $\underline{R} = \inf\mathcal{U}$, the value of the debt claim is less than the outside option, $1 + \bar{r}$. Second, at $\bar{R} = \sup\mathcal{U}$, the value of the debt claim is greater than $1 + \bar{r}$. And third, that the value of the debt claim is strictly increasing in the repayment.

The first criteria is trivially satisfied, given the definition of $V(h,R)$ and since $\underline{R}=1+\bar{r}$. For the second criteria, suppose that $V_h<0$. We verify this claim later. In that case,

$$\begin{aligned} V(1,\bar{\theta}) &= \frac{\bar{R}}{\bar{\theta}-\underline{\theta}}\Bigg\{\bar{\theta}-\underline{\theta}-(1-\phi)^n\left[\frac{\bar{R}}{\sigma\delta}-\underline{\theta}\right] \\ &- \left[\frac{\bar{R}(1-\phi)}{\sigma\delta+\eta}-\underline{\theta}\right]\left(1-(1-\phi)^n+\int_0^{\phi} ldF_1(\ell)\right)\Bigg\} \\ &> \Xi \equiv \frac{\bar{R}}{\bar{\theta}-\underline{\theta}}\left\{\bar{\theta}-\underline{\theta}-\left[\frac{\bar{R}}{\sigma\delta}-\underline{\theta}\right](1+\phi)\right\}. \end{aligned}$$

Insofar $\bar{r}<\underset{\sim}{r}\equiv\Xi-1$, then the second criteria is satisfied as well. Finally, to show that the claim is increasing in the repayment, note that

$$\begin{aligned} V_R &= \frac{1}{\bar{\theta}-\underline{\theta}}\Bigg\{\bar{\theta}-\underline{\theta}-(1-\ell^*(h))^n\big(2\theta^*(h)-\underline{\theta}\big)h \\ &- \big(1-(1-\ell^*(h))^n\big)\big(2\theta^{**}-\underline{\theta}\big)\left(1-\phi+\int_0^{\ell^*(h)} ldF_1(l)\right)\Bigg\}. \end{aligned}$$

The terms in the curly brackets are greater than

$$\bar{\theta}-\underline{\theta}-\big(2\theta^*(h)-\underline{\theta}\big)\left\{1-\phi+(1-\ell^*(h))^n\ell^*(h)+\ell^*(h)\right\},$$

which is decreasing in the haircut. Therefore a sufficient condition for $V_R>0$ is

$$\bar{\theta}>\tilde{\theta}\equiv\underline{\theta}+\big(2\theta^*(1)-\underline{\theta}\big)\left(1+\phi(1-\phi))^n\right).$$

Comparative statics. We determine how the repayment depends on the haircut, h, output distribution, $\bar{\theta}$, creditor-friendliness, ϕ, and holdout cost, η. To derive the effects on repayment, we first need to sign the effects on the implicit function,

$V(h,R)$:

$$\begin{aligned}
V_h &= \frac{R}{\bar{\theta}-\underline{\theta}}\Big\{\big(\theta^*(h)-\theta^{**}\big)\Big[n\big(1-\ell^*(h)\big)^{n-1}h-\big(1-\ell^*(h)\big)^n\Big] \\
&- \big(\theta^{**}-\underline{\theta}\big)n\big(1-\ell^*(h)\big)^{n-1}h-\theta^*(h)\big(1-\ell^*(h)\big)^n\Big\}<0 \\
V_{\bar{\theta}} &= \frac{R-V}{\bar{\theta}-\underline{\theta}}>0 \\
V_\eta &= \frac{R}{\bar{\theta}-\underline{\theta}}\left\{\big(1-(1-\ell^*(h))^n\big)\frac{\theta^{**}(1-\phi)}{\sigma\delta+\eta}+\frac{\theta^{**}}{\sigma\delta+\eta}\int_0^{\ell^*(h)} l dF_1(l)\right\}>0 \\
V_\phi &= \frac{R}{\bar{\theta}-\underline{\theta}}\left\{\big(1-(1-\ell^*(h))^n\big)\big(2\theta^{**}-\underline{\theta}\big)+\frac{R}{\sigma\delta+\eta}\int_0^{\ell^*(h)} l dF_1(l)\right\}>0
\end{aligned}$$

For $V_h<0$, we impose that $\underline{\theta}<\underset{\sim}{\theta}$, which is defined in the proof of Proposition 3. It follows from the implicit function theorem that

$$\frac{\partial R^*}{\partial h}>0\,, \quad \frac{\partial R^*}{\partial \bar{\theta}}<0\,, \quad \frac{\partial R^*}{\partial \eta}<0\,, \quad \text{and} \quad \frac{\partial R^*}{\partial \phi}<0\,.$$

The results in Proposition 5 follow directly from applying the results of Propositions 3 and 4 to determine how the curves $h^*(R)$ and $R^*(h)$ shift following changes in the underlying parameters.