Sovereign risk and bank fragility*

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Abstract

We develop a model of bank risk-taking with strategic sovereign default. Domestic banks invest in real projects and purchase government bonds. While an increase in bond purchases crowds out profitable investments, it improves the government's incentives to repay and therefore lowers its borrowing costs. For low levels of government debt, banks influence their default risks through purchases of bonds. But, for high debt levels, this influence is lost since bank and government default are perfectly correlated. Banks fail to account for how their bond purchases influence the government's default incentives. This leads to socially inefficient levels of bond holdings.

Keywords: sovereign debt, financial intermediation, financial repression, bank fragility.

JEL classifications: G01, G21, G28.

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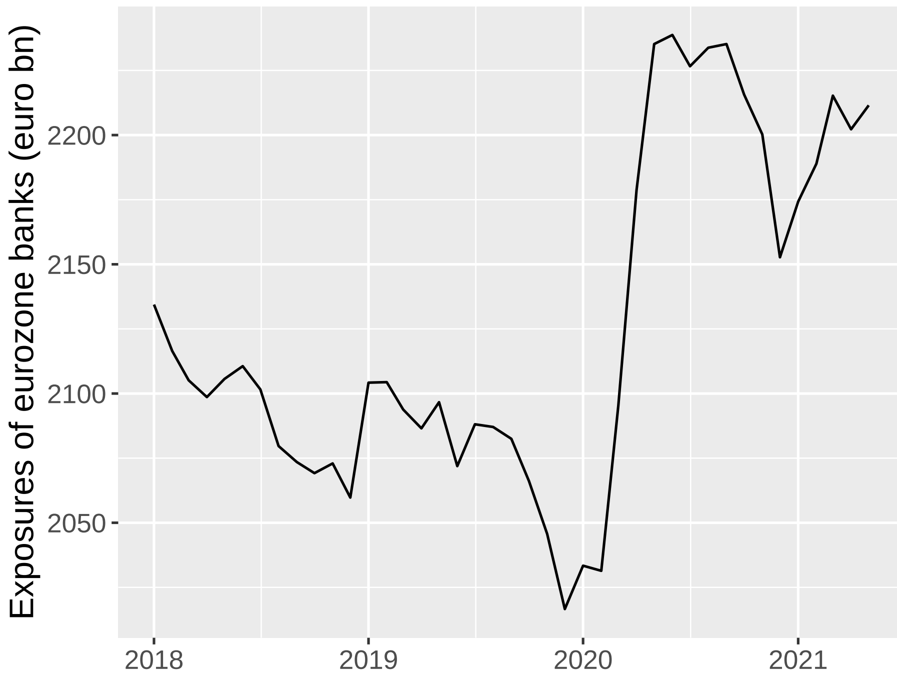
1 Introduction

The Covid-19 pandemic has placed great burdens on governments' coffers, with many issuing record-breaking levels of debt. This, in turn, has lead to a sharp increase in banks' holdings of domestic sovereign debt, especially within the euro-area (Figure 1), and rekindled concerns over the sovereign-bank "doom-loop" and financial stability (Schnabel, 2021). Open questions from the euro-area sovereign debt crisis of 2012 on how sovereign default risk shapes banks' risk-taking, and whether banks' holdings of government bonds should be regulated have been brought back to the fore. Then, as now, it was feared that increases in banks' holdings of domestic sovereign debt would crowd out domestic investment (Brunnermeier et al., 2016; Acharya et al., 2018). But, at the same time, this would help stabilise financial markets and thereby limit the rise in sovereign spreads (Asonuma et al., 2015).

In this paper, we develop a model of bank risk-taking with strategic sovereign default risk to answer these questions. Competitive domestic banks, subject to limited liability, decide between purchasing government bonds issued by the local government to finance public debt, and investing in the real economy. Foreign investors also purchase government bonds. The government, which only cares about domestic welfare, subsequently chooses to either repay or default. While repaying involves transferring resources to foreign investors, defaulting results in deadweight losses on the economy. Thus, the government is more likely to repay if domestic banks hold more government bonds. But, the associated crowding-out of investment reduces the tax base, which dampens the government's willingness to repay.

Our first result is that the connection, or '*nexus*', between bank default risk and

Figure 1: Exposures of eurozone banks to domestic government debt



This figure shows the exposure of eurozone banks to domestic government debt and loans. Following the outbreak of the Covid-19 pandemic in 2020, banks sharply increased their exposures (*Source*: ECB).

sovereign default risk depends crucially on the level of public debt. An '*asymmetric nexus*', which arises for low levels of debt, is characterised by banks defaulting whenever the sovereign defaults, but not vice versa. As such, sovereign default risk is lower than bank default risk. While in the '*symmetric nexus*', which is obtained for higher levels of debt, bank default and sovereign default are perfectly synchronised. In this case, which resembles events during the European sovereign debt crises period, the default risks for banks and sovereigns coincide.

A second result is that in the asymmetric nexus, an increase in a bank's holdings of government bonds reduces its own likelihood to default since bonds are relatively less risky. Thus, the bank's optimal portfolio trades-off reducing the likelihood to default versus achieving higher returns while subject to limited-liability. Under the symmetric nexus, however, a bank's likelihood to default is identical to that for the government and cannot be altered by marginal changes in the bank's portfolio. The intuition for this result is as follows. If the government marginally prefers to repay over defaulting, then all banks have strictly positive equity values. But, if the government prefers to default over repaying, then banks' equity values are zero. Thus, each bank's equity value is discontinuous at the point where the government is indifferent between repaying and defaulting.

The third result concerns how changes to bank capital influences bank risk-taking and sovereign default risk. In general, the effect of an increase in capital on banks' portfolios can be decomposed into two effects. First, since banks have more to loose, they prefer to make safe investments. This '*skin in the game*' effect encourages banks to purchase more bonds. And second, there is an indirect '*general equilibrium*' effect, which relaxes the crowding-out of investment, leads to greater investment. Under the asymmetric nexus, the direct effect dominates the indirect effect, and so banks' investments decline as capital increases. While, under the symmetric nexus, only the indirect effect is present and so banks' investments increase with capital. The general equilibrium effect also underpins how sovereign default risk is influenced by changes in bank capital: as better capitalised banks invest more in the real economy, this increases domestic output, which strengthens the government's incentives to repay.

Our fourth result shows that the competitive equilibrium is generically inefficient – banks hold either too much or too few domestic government bonds. Our welfare criterion is constrained efficiency: the social planner chooses the banks' portfolios to maximise aggregate domestic welfare while accounting for the portfolios impact on the government's default incentives. If the dead-weight loss on the economy from the government defaulting is high, then there is under-investment in domestic government bonds. In such situations, policies aimed at limiting banks' holdings of government bonds are welfare reducing. In contrast, policies that encourage banks to increase their holdings of government bonds improve welfare. In what follows, we refer to such policies and bank regulation constituting a form of '*financial repression*'.[1]

Conversely, if the dead-weight loss of a default is low, there is over-investment in government bonds. In this case, limiting bank's holdings of government bonds improves welfare. Importantly, the critical dead-weight loss at which financial repression becomes optimal depends on the nexus. As such, there is a range of values for the dead-weight loss for which financial repression is optimal under the asymmetric nexus, while it is sub-optimal in the symmetric nexus.

Our model has two implications for the interpretation of the European sovereign debt crisis. First, the observed increase in banks' holdings of domestic government bonds in stressed countries can be viewed as a market outcome. This risk-taking perspective to explain the observed outcomes is consistent with that findings of Acharya and Steffen (2015) and others. '*Moral suasion*', i.e., informal government pressure on domestic banks to buy more domestic government bonds might have played an additional role (e.g., Ongena et al., 2019) but is not required to explain the observed developments. Second, even if there was moral suasion, our normative results suggest that this may have improved welfare.

[1] The term dates back to the work of McKinnon (1973) and Shaw (1973) and is used to capture a range of policies that redirect private capital to governments.

Our results also inform the current debate on regulating banks' holdings of domestic government bonds. First, we show that history matters. The desirability of limiting banks' exposure to sovereign debt depends on the type of the nexus which, in turn, depends on the amount of outstanding debt. Second, our results also show that the size of default costs are crucial. Limiting banks' exposure is welfare improving, in particular, when sovereign default costs are low. But, the contrary holds too: limiting banks' exposure is welfare reducing when sovereign default costs are high. Thus, regulations that limit banks' holdings of domestic government bonds should be accompanied by introducing measures that reduce the costs of sovereign defaults.

Related literature. Our paper relates to the growing theoretical literature on sovereign risk and bank risk-taking (see e.g., Ari, 2018 and Crosignani, 2021).[2] These papers find that riskier banks tend to buy more risky domestic government bonds because of their limited liability status. These papers, however assumes that sovereign risk is exogenous and non-strategic. We depart by considering how strategic sovereign default interacts with bank risk-taking.[3]

Uhlig (2013) and Farhi and Tirole (2017) consider how banking supervision can influence banks' risk-taking in the presence of sovereign default risk. Banks load up on risky domestic government bonds because of lax domestic financial supervision. We show that banks may load up on domestic government bonds when it is in their

[2]Other theoretical contributions on on the sovereign-bank nexus include König et al. (2014), Cooper and Nikolov (2018) and Leonello (2018). While these papers focus on the role of government guarantees in propagating risks, we focus on how banks' holdings of sovereign bonds influence strategic sovereign default.

[3]Our result on the synchronicity between the bank and government default thresholds in the symmetric nexus shares a family resemblance with results in Allen et al. (2015) and Gale and Gottardi (2020) on how banks and firms align their bankruptcies. An important driver behind the similarity in the results is the segmentation of funding markets.

(private) interest to do so. Limited liability implies that they typically do not care about states of the world in which the government defaults since in these states they default as well. However, we also show that in some states of the world, banks hold too few government bonds. In such situations, laxer supervision than usual might be one way to get closer to the social optimum.

Our normative result on the appropriateness of financial repression stems from a pecuniary externality: banks do not internalise the effect of their portfolios on the price of sovereign bonds. In related work, Chari et al. (2020) develop a model of optimal financial repression in a closed economy. In their model financial repression is optimal only when the government faces large refinancing needs. Since they focus on a closed economy, the benefit that lowering the interest rate on sovereign debt leads to a lower outflow of tax revenue if the government chooses to repay, is absent in their model. A further difference is that banks in our model enjoy limited liability which sometimes induces them to hold too much sovereign debt. This happens when the crowding-out of real investments, and therefore future tax revenue, is relatively larger than the benefit that a lower interest rate on government bonds provides.

Our paper also contributes to the literature on the costs of sovereign default. Gennaioli et al. (2014) present a model where banks hold government bonds to store liquidity for future investments. As such, a government default dries up liquidity in the banking sector, thereby reducing credit and output. In our model, banks hold government bonds for investment purposes. We, thus, explore how bank risk-taking influences sovereign default risk. Broner et al. (2014) argue that even if a sovereign could perfectly discriminate between defaulting on foreign bondholders but not on domestic ones, the full costs of a sovereign default will be borne by domestic bondholders who buy bonds from foreign bondholders in a secondary market. In our model, default is non-discriminatory and impacts both domestic and foreign bondholders.

The remainder of the paper is organised as follows. Section 2 describes the model and Section 3.1 derives the equilibrium and testable hypotheses. In Section 4 we extend our model and determine the social optimum. In Section 5 we contextualise our normative results within the recent policy debates on regulating banks' holdings of sovereign debt. A final section concludes. All proofs are relegated to the appendix.

2 Model environment

We now present our model to explore how endogenous sovereign default risk shapes bank risk-taking. There are two dates, $t = 0$ and $t = 1$ and a single perishable good that is used for both consumption and investment. The economy consists of 'domestic' and 'foreign' agents, all of whom care about consuming at $t = 1$. Domestic agents include distinct unit masses of risk-neutral bankers and infinitely risk-averse households. In addition, a domestic government is responsible for insuring households' deposits, repaying bond holders and providing a public good. It chooses its policies to maximise aggregate domestic welfare. Foreign agents consist of a large pool of risk-neutral investors. The only source of uncertainty is an aggregate shock, $A \geq 0$, that is realized at $t = 1$.

An important assumption in our setup is that domestic and foreign capital markets are segregated. As such, foreign investors cannot hold deposits in domestic banks and domestic bankers and households cannot invest abroad. A link between the two markets is, nevertheless, provided by the government who issues bonds to all domestic and foreign agents.

Domestic bankers. The representative domestic banker owns and operates a domestic bank. All domestic banks are identical, operate under perfect competition and enjoy limited liability. The banker is endowed with $k > 0$ at $t = 0$, which is invested as bank equity. The representative banker's utility function is $U^B = G/2 + c_1$, where $c_1 \geq 0$ is the bank equity value and $G \geq 0$ is the level of the public good provided by the government, which is shared by all domestic agents.

The bank borrows $h > 0$ from households at $t = 0$ by issuing one-period debt contracts (deposits) that carry an interest rate $r_d > 0$. The bank can invest $\ell \leq k+h$ in a project (real economy) at $t = 0$ that yields $A\ell^\alpha$ at $t = 1$, where $\alpha < 1$. The aggregate shock, $A \geq 0$, is a random variable drawn at the start of $t = 1$, that is common for all banks. It is distributed according to the known cumulative distribution function $F(A)$. We denote the corresponding probability distribution function by $f(A)$. The bank can also purchase $b \equiv k + h - \ell \leq 0$ worth of government bonds at $t = 0$ with a gross return of $(1 + r_g)$ at $t = 1$ if the government repays and 0 if the government defaults.[4]

The bank repays depositors in full at $t = 1$ if the returns from investing in the real economy and purchasing government bonds are sufficiently high. But, if the returns are low, the bank defaults. In this event, all of the bank's resources are transferred to the depositors and the bank's equity value is zero.

[4] We abstract from the role of sovereign debt restructuring, which would generate a positive repayment even if the government defaults. However, this would not qualitatively alter our results.

Domestic households. The representative domestic household is endowed with $d > 0$ of the consumption good. At $t = 0$, the household invests $h \leq d$ in insured bank deposits and the remainder, $d - h$, in domestic government bonds. The $t = 1$ utility function for the representative domestic household's utility function is $U^H = G/2 + \min_{\{A\}} c_1$, where $c_1 \geq 0$ are the accrued returns, which depends on the aggregate shock. Thus, households' risk-aversion only directly influences their private consumption, while the level of public good provision by the government is taken as a given.

Domestic government. At $t = 0$, the government has a stock, $S > 0$, of legacy debt that needs to be refinanced. To this end, the government issues an infinitely divisible one-period bond with face value $S(1 + r_g)$, where r_g is the endogenous net interest rate. At the same time, the government decides whether or not to insure bank deposits.

At $t = 1$, the government is endowed with $T > 1$, has powers to tax households' private consumption and chooses to either default or repay bond holders. Default is non-discriminatory and so both foreign and domestic agents suffer losses on their bond holdings. In particular, the losses suffered by the domestic bank impair its ability to adequately manage projects, thereby reducing project returns by a fraction $\delta \leq 1$. We offer two possible explanations for this assumption. First, banks use government bonds and other liquid assets to manage credit lines for firms. Following the government default, banks are unable to service the credit lines, which hamper the real economy (Bofondi et al., 2017). And second, insofar that the losses borne by the bank following the government default reduce its charter value, this increase the scope for shirking or absconding by the banker (Keeley, 1990; Calomiris and Kahn, 1991). These actions, in turn, further reduce the value of the bank's investments.

The cumulative losses suffered by domestic banks impinge on their abilities to repay depositors. The government can insulate depositors from losses by credibly insuring their deposits. This is achieved by encumbering a portion of the endowment, T, for deposit insurance.[5] The remainder – after paying for deposit insurance – along with additional tax revenue raised from households, can be used to repay bond holders. Anything that is left over constitutes the public good provided by the government.[6]

Foreign investors. Foreign investors are deep-pocketed. At $t = 0$, the representative investor can either purchase government bonds or invest in the world capital market at rate $\bar{r} > 0$.

Timing. At $t = 0$, the government issues bonds and chooses whether to insure bank deposits or not; domestic banks, domestic households and foreign investors choose how much of the government bond to purchase; domestic banks issue deposits to households and invest in projects. At $t = 1$, the aggregate shock, A, is realised; the government chooses whether to repay or default on its debts; banks either repay households in full or default and are protected by limited liability; the government provides the public good; domestic bankers, domestic households, and foreign in-

[5]We, thus, argue that domestic depositors are senior claimants on the government's resources. This line of reasoning can be motivated by appealing to political economy considerations where domestic depositors might vote out an incumbent government during an election if they suffer large losses (Rosenbluth and Schaap, 2003).

[6]This model environment allows us to side-step the issue of pricing of deposits as we show in Section 3.1.1. While such an exercise could be done, for example, along the lines of Carletti et al. (2020), this would greatly complicate the model and is not central to our analysis.

vestors consume.

3 Equilibrium

Before solving the full model, let us consider some simpler benchmarks that allow us to better articulate how the different frictions influence the outcome.

First-best. To start with, suppose that sovereign default risk is exogenous, the bank is not subject to limited liability and that there is no deposit insurance. Denoting the government's (exogenous) failure threshold by $\widehat{A}_S$, the bank's optimal, and the first-best level of investment is given by $\ell^{FB} = \arg\max_\ell \int_0^\infty AdF(A)\,\ell^\alpha + b(1+r_g)$, subject to $b = k - \ell$ and that government bonds are priced according to $\big(1 - F(\widehat{A}_S)\big)(1+r_g) = 1+\bar{r}$. Since there is no deposit insurance, infinitely rise-averse households do not hold deposits and so the bank can only invest up to the level of its capital. We thus obtain

$$\ell^{FB} = \left(1 - F(\widehat{A}_S)\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha \int_0^\infty AdF(A)}{1+\bar{r}}\right)^{\frac{1}{1-\alpha}}. \tag{1}$$

The bank invests up to the point that the marginal project returns are equal to the expected return from holding government bonds.

Exogenous sovereign risk. Next, suppose the government introduces deposit insurance and that the bank is subject to limited liability. Consequently, households deposit their entire endowments with the bank. Normalizing the return on deposits to zero, we get that, conditional on the government repaying all bond holders, the bank defaults whenever $A < \widehat{A}_B \equiv \frac{d-(1+r_g)b}{\ell^\alpha}$. Insofar that $\widehat{A}_B > \widehat{A}_S$, the bank's optimal investment is given by $\ell^* = \arg\max_\ell \int_{\widehat{A}_B}^{\infty} \left[A\ell^\alpha + b(1+r_g) - d\right] dF(A)$, subject to the balance sheet condition, $b = d + k - \ell$ and the pricing of government bonds. We obtain

$$\ell^* = \left(\frac{1 - F(\widehat{A}_S)}{1 - F(\widehat{A}_B)}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha \int_{\widehat{A}_B}^{\infty} A dF(A)}{1+\bar{r}}\right)^{\frac{1}{1-\alpha}} . \tag{2}$$

In the limit where bank and government failure perfectly coincide, $\widehat{A}_S \to \widehat{A}_B$, we have that the level of investment is strictly lower than under the first-best allocation, i.e., $\lim_{\widehat{A}_S \to \widehat{A}_B} \ell^* < \ell^{FB}$. Thus, by distorting the bank's incentives to focus only on the upside of asset returns, the introduction of limited liability and deposit insurance lead to the bank holding more government bonds in equilibrium. But, when there is a wedge between the bank's and government's failure threshold, the comparison is ambiguous and depends on the size of the wedge. In what follows, we explore this further by solving the competitive equilibrium for the full model where sovereign risk is endogenous and depends on the bank's investment decisions.

3.1 Competitive equilibrium

We solve the model by backward induction.

Definition 1. *The symmetric pure-strategy sub-game perfect equilibrium comprises of: (i) the representative bank's allocation between purchasing government bonds and investing in the project, $\{b^*, \ell^*\}$, the interest rate on deposits, r_d^*, and a critical default threshold, $\widehat{A}_B^*$; (ii) the representative household's allocation between bank deposits and government bonds, $\{h^*, d - h^*\}$, and (iii) the interest rate that the government must pay to roll over its debt, r_g^*, and a critical default threshold, $\widehat{A}_S^*$, such that*

1. *At $t = 1$, the government repays whenever $A \geq \widehat{A}_S^*$, given bank's and household's*

allocations and interest rates on government bonds and bank deposits.

2. *At $t = 1$, the bank repays whenever $A \geq \widehat{A}^*_B$, given the government decision, the bank's and household's allocations and the interest rates.*

3. *At $t = 0$, the bank and household choose their allocations, $\{b^*, \ell^*\}$ and $\{h^*, d - h^*\}$, respectively, given the bank's and government's default thresholds and interest rates.*

4. *At $t = 0$, foreign investors set r^*_g from their participation constraint and the bank sets the interest rate, r^*_d on deposits.*

In what follows, we first solve for the interest rates that the bank offer to households, and subsequently use this result to derive the bank's and the government's default thresholds.

3.1.1 Interest rate on bank deposits

The representative household chooses between bank deposits and purchasing government bonds. But, both options are inherently risky where, in the worst case, both banks and government default on their obligations to the household at $t = 1$. Thus, in the absence of a credible deposit guarantee by the government, households are indifferent between lending to banks, purchasing government bonds and autarky. By ensuring that bank deposits are safe, the guarantee induces households to strictly prefer lending to banks, which increases the overall level of investments.

For the government to credibly provide the deposit guarantee, we require that households are senior claimants on the government's resources. Since $T > 1$, the government can fully guarantee households' initial deposits.[7] Thus, households bear no risk from lending to banks. And, since bonds are subject to default risk, households prefer to deposit their entire endowment with banks. Finally, insofar that only the principal is insured and households and infinitely risk-averse and only value safety, banks offer deposit contracts with a zero interest rate due to perfect competition. Lemma 1 summarises.

Lemma 1. *With a credible government guarantee on households' deposits, the equilibrium deposit rate is $r_d^* = 0$. The representative household invests its entire endowment in bank deposits, i.e., $h^* = d$.*

It is worth noting that the result of Lemma 1 would also obtain in an environment where households are risk-neutral and banks are local monopolies over subsets of households. Thus, while banks cannot extract full monopoly rents, they would nevertheless continue to set $r_d^* = 0$ to extract wealth from local households.

3.1.2 Government default

Following the realisation of the aggregate shock, A, at $t = 1$, suppose that the government chooses to repay bond holders. The equity value of the representative bank is given by $\overline{e} \equiv \max\{0,\, A\,\ell^{\alpha} + (1 + r_g)\,b \,-\, d\}$, and the bank defaults whenever $A < \widehat{A}_B \equiv \bar{A} = \frac{d-(1+r_g)b}{\ell^{\alpha}}$. Thus, after paying the deposit insurance, the government has revenue $\overline{R} \equiv T - \max\{0,\, d - A\,\ell^{\alpha} - (1 + r_g)\,b\}$ remaining.

If $\overline{R} \geq S(1 + r_g)$, then the government pays bond holders using the revenue

[7] Implicitly, we assume that the government does not need to finance the guarantee by issuing additional external debt, as in Farhi and Tirole (2017), but can manage the payments using internal resources.

and provides $\overline{G} = \overline{R} - S(1 + r_g)$ towards the public good. The representative banker and household obtain utilities $U^B = \overline{G}/2 + \overline{e}$ and $U^H = \overline{G}/2 + d$, respectively. Alternatively, if $\overline{R} < S(1+r_g)$, then the government taxes households at the rate $\tau = \frac{S(1+r_g)-\overline{R}}{d}$ and pays bond holders using the combined revenue and taxes. Moreover, the government is unable to provide the public good. The utilities of the representative banker and household are $U^B = \overline{e}$ and $U^H = d(1-\tau) = d-\big(S(1+r_g)-\overline{R}\big)$. Irrespective of how the repayment of bond holders is financed, we obtain that aggregate utility of domestic bankers and households is given by

$$V^R(A) \equiv T + A\ell^\alpha - \big(S - b\big)(1 + r_g)\,. \tag{3}$$

Suppose, instead, that the government decides to default on bond holders. In this case, the bank's equity value is $\widetilde{e} = \max\{0,\, (1-\delta)\,A\,\ell^\alpha - d\}$ and the bank defaults whenever $A < \widehat{A}_B \equiv \widetilde{A} = \frac{d}{(1-\delta)\ell^\alpha}$. Since the government default leads to losses on both bonds purchased and investments, the bank is more likely to fail whenever the government defaults. This implies an ordering of the two bank default thresholds whereby $\bar{A} < \widetilde{A}$. In Section 3.1.4, we show how the relationship between these thresholds and that for the government play an important role in determining the equilibrium.

Government revenue, after paying deposit insurance, is given by $\widetilde{R} = T - \max\{0,\, d - (1-\delta)\,A\,\ell^\alpha\}$. Since the government has no further obligations, this amount is used in its entirety to provide $\widetilde{G} = \widetilde{R}$ worth of the public good. The utilities of the representative banker and household are $U^B = \widetilde{G}/2 + \widetilde{e}$ an $U^H = \widetilde{G}/2 + d$, respectively. Aggregate utility of domestic bankers and households is

$$V^D(A) \equiv T + (1 - \delta)\, A\, \ell^\alpha \,. \tag{4}$$

Comparing the levels of aggregate domestic utility between defaulting and repaying, the government repays whenever

$$A \geq \widehat{A}_S \equiv \frac{(S - b)(1 + r_g)}{\delta\, \ell^\alpha} \,. \tag{5}$$

By choosing to repay, the government splits $S(1 + r_g)$ worth of domestic resources proportionally between domestic banks and foreign investors based on their holdings of government bonds. As the amount that accrues to the foreign investors, i.e., the numerator in Equation (5), increases, aggregate domestic domestic utility is reduced.

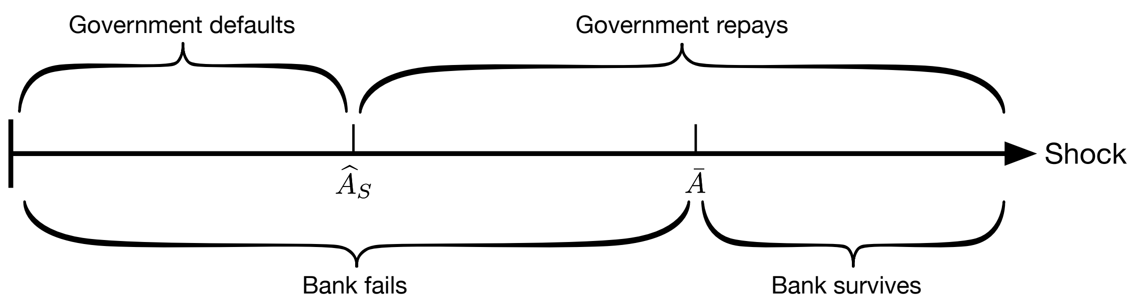
By defaulting, the government does not raise taxes to repay foreign investors and domestic banks. Moreover, banks suffer losses on their investments due to the deadweight losses suffered by the domestic economy. These losses to banks' investments are captured by the denominator in Equation (5). Thus, the government repays bond holders whenever the reduction to aggregate domestic utility from resources accruing to foreign investors is smaller than the banks' losses if the government defaults.

Next, we solve for the representative bank's portfolio allocation and determine the interest rate on government bonds. We treat each in turn.

3.1.3 Bank's optimal portfolio

At $t = 0$, the representative bank chooses how much to invest in the real economy and how many government bonds to purchase. Due to perfect competition, the bank acts as a price taker in the market for government bonds and therefore does not internalise how changes in its bond holdings influences the government's default incentives. Nevertheless, sovereign default risk shapes the bank's incentives via the position of the government's default threshold, $\widehat{A}_S$, relative to those for the bank. We distinguish between two cases.

Figure 2: Asymmetric nexus.



This figure shows the case of an asymmetric nexus, where the bank always fails when the government defaults but not vice versa.

Case 1. Asymmetric nexus $\left(\widehat{A}_S < \bar{A} < \widetilde{A}\right)$**.** If the government defaults, $A < \widehat{A}_S$, then the bank also defaults. But, if the government repays, $A \geq \widehat{A}_S$, then the bank is able to repay depositors in full and retain a positive equity value as long as $A \geq \bar{A}$. Thus, if the aggregate shock lies in the interval $(\widehat{A}_S, \bar{A})$, then the bank defaults even though the government repays all bond holders. Since the bank defaults for a larger range of shocks than the government, ex-ante bank default risk is greater than that for the government. Figure 2 depicts the classification of default thresholds under the asymmetric nexus. Consequently, the bank's portfolio problem is

$$\max_{\ell,b} \quad \int_0^\infty \bar{e}(A)\, dF(A) = \int_{\bar{A}(\ell,b)}^\infty \Big(A\,\ell^\alpha \;+\; (1+r_g)\,b \;-\; d \Big)\, dF(A)\,,$$

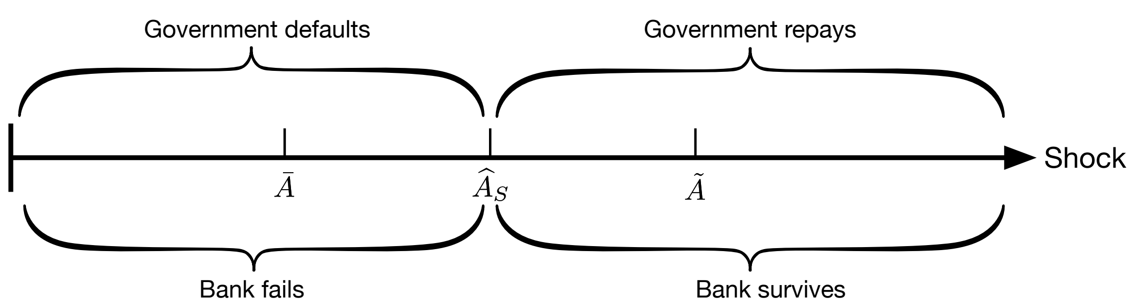
subject to the balance sheet constraint $\ell + b = d + k$.

Case 2. Symmetric nexus $\big(\bar{A} < \widehat{A}_S < \widetilde{A}\big)$**.** If the government defaults, $A < \widehat{A}_S$, then the bank also defaults because $\widehat{A}_S < \widetilde{A}$. But, whenever the government repays, $A \geq \widehat{A}_S$, it follows that the bank has a strictly positive equity value and repays households since $\bar{A} < \widehat{A}_S$. Figure 3 depicts the default thresholds under the symmetric nexus. In its optimisation problem, the bank, effectively, replaces its own default threshold with that of the government and the bank's portfolio problem is

$$\max_{\ell,b} \quad \int_0^\infty \mathbb{1}_{A>\widehat{A}_S}\, \bar{e}(A)\, dF(A) = \int_{\widehat{A}_S}^\infty \Big(A\,\ell^\alpha \;+\; (1+r_g)\,b \;-\; d \Big)\, dF(A)\,,$$

subject to the balance sheet constraint. Since bank and government default are perfectly synchronised, they are both equally risky.

Figure 3: Symmetric nexus.



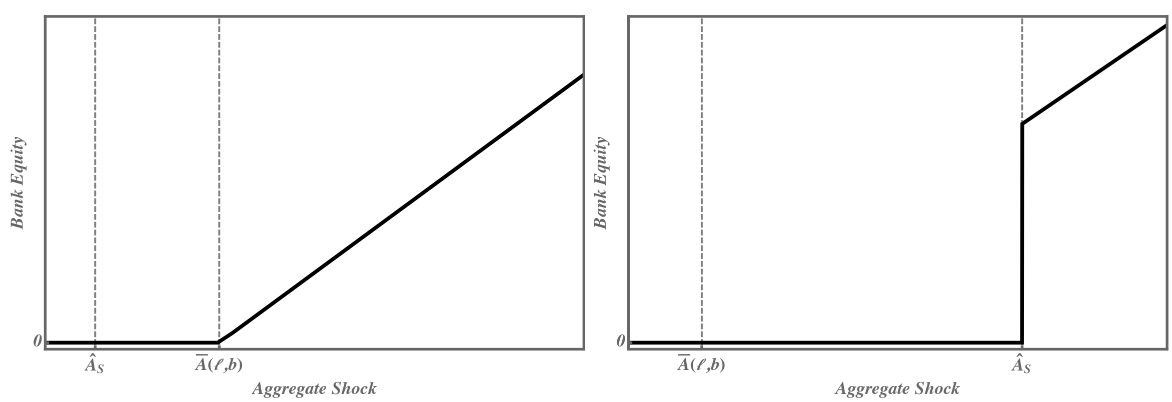
This figure shows the case of a symmetric nexus, where bank and government always fail at the same time.

Figure 4 plots bank equity value under the two cases. For the asymmetric nexus, the equity value is convex in the aggregate shock wherein the limited liability constraints binds for $A < \bar{A}(\ell, b)$. As such, a small change in the aggregate shock always leads to small changes in bank equity value. Moreover, by changing its investment decision, the bank can shift its failure threshold. Thus, in equilibrium, the bank's optimal investment choice trades-off attaining higher returns versus reducing fragility.

In the symmetric nexus case, however, equity value is strictly positive for $A \geq \widehat{A}_S$ and zero otherwise. Importantly, there is a discontinuous jump at the government's default threshold, which is the de facto failure threshold for the bank. As such, the bank is unable to influence its failure threshold via its investment decision.

Figure 4: Bank equity value under the asymmetric nexus and symmetric nexus.

(a) Asymmetric nexus (b) Symmetric nexus



For the asymmetric (symmetric) nexus case, we have $S = 0.35$ ($S = 0.7$). All other parameters are the same in both cases: $d = 0.5$, $k = 0.5$, $\delta = 0.9$, $\alpha = 0.4$ and $\bar{r} = 0$. The aggregate shock follows an exponential distribution with hazard rate $\lambda = 0.2$.

In principle, there is also a third case to consider where bank and government default are asynchronous and the ordering of thresholds satisfies $\bar{A} < \tilde{A} < \widehat{A}_S$. If the government repays, $A \geq \widehat{A}_S$, then the bank would always repay since $A \geq \bar{A}$. But, when the government default, $A < \widehat{A}_S$, there are two possibilities depending on the size of the shock. If $A < \tilde{A}$, then the bank would also default. But, if $A > \tilde{A}$, then the bank would repay, Moreover, its equity value would jump from $\delta A\ell^{\alpha} - d$ to $A\ell^{\alpha} + (1 + r_g)b - d$ at the government's default threshold, $\widehat{A}_S$. However, as we shall argue, this government default threshold is associated with a far too high interest rate charged by foreign investors that the debt is never re-financed at $t = 0$ and there is market breakdown. Thus, this case is not material in equilibrium.

3.1.4 Interest rate on government bonds

Focusing on equilibria where foreign investors are marginal buyers of government bonds, the interest rate, r_g, is determined according to their binding participation constraint, i.e.,

$$\big(1 - F(\widehat{A}_S)\big)\big(1 + r_g\big) = 1 + \bar{r}\,. \tag{6}$$

To characterise the equilibrium, we make the following two assumptions.

Assumption 1. *The hazard rate of the aggregate shock distribution λ is constant such that the government's propensity to default is relatively large, i.e $\lambda > \widehat{\lambda}$ where the threshold is formally defined in Appendix A.*

With a constant hazard rate, we are better able to isolate how changes in the bank's portfolio influence the government's default incentives, and how this translates into the pricing of government bonds.[8] And assuming a lower bound for the hazard rate ensures that, once government debt grows beyond a level that sustains

[8]If the hazard rate is not constant, then a marginal change in the bank's portfolio that influence's the government's incentives to repay also induces a marginal change in the hazard rate. Insofar that the hazard rate is increasing – as is the case for a Normal distribution as well as for a Log-normal distribution with a non-negative mean – this effect exacerbates the original incentive effect without qualitatively altering the mechanism.

either the asymmetric or symmetric nexus, foreign investors charge exorbitantly high interest rates, which the government can never hope to repay. And so there is market breakdown.

Assumption 2. *The bank is awash in funding, i.e.,* $S < d + k$.

This ensures that an increase in the bank's holdings of government bonds reduces the government's incentives to default, i.e., $\frac{\partial \widehat{A}}{\partial \ell} > 0$. Moreover, the influence of the bank's holdings of government bonds on the government's default incentives remain robust to the introduction of a domestic non-bank financial sector (e.g., pension and insurance sector) that also holds government bonds. For example, if this sector holds a stock $N > 0$ of government bonds, the government's default threshold is given by $\widehat{A}_S = \frac{(S-N-b)(1+r_g)}{\delta \ell^{\alpha}}$. Accounting for the bank's balance sheet, $\frac{\partial \widehat{A}}{\partial \ell} > 0$ for all $N \geq 0$.

It is well established that in such models, where governments lack the ability to commit on a policy of always repaying bond holders, multiple equilibria arise and are driven by investors' beliefs (Calvo, 1988). If investors believe that the government will repay, the required return on bonds is low, which the government can readily service, reducing the incentives to default. While, if investors believe that the government will default, then the required return is high, which makes it more likely that the government will default. The equilibrium where investors believe that the government will repay is Pareto efficient and the focus of our analysis. Proposition 1 describes the resulting equilibrium allocations.

Proposition 1. *There exist unique bounds,* $\underline{S}$, *and* $\bar{S}$ *on the level of government debt, where* $\underline{S} < \bar{S}$, *such that:*

- *For* $S \leq \underline{S}$ *the equilibrium is characterised by the asymmetric nexus where the* *bank's investment in real projects is given by*

$$\ell^* = \left(\frac{1 - F(\widehat{A}_S)}{1 - F(\bar{A}(\ell^*))} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha \int_{\bar{A}(\ell^*)}^{\infty} A \, \mathrm{d}F(A)}{1 + \bar{r}} \right)^{\frac{1}{1-\alpha}} . \tag{7}$$

Purchases of government bonds is given by $b^* = k + d - \ell$ *and the sovereign's default threshold is implicitly defined by* $\tau(\widehat{A}_S^*) = 0$*, where* $\tau(\widehat{A}_S) \equiv \widehat{A}_S - \frac{S-b}{\delta \ell^\alpha} \left(\frac{1+\bar{r}}{1-F(\widehat{A}_S^*)} \right)$.

- *For* $\underline{S} < S \leq \bar{S}$*, the equilibrium is characterised by the symmetric nexus where the bank's investment in real projects is given by*

$$\ell^* = \left(\frac{\alpha \int_{\widehat{A}_S}^{\infty} A \, \mathrm{d}F(A)}{1 + \bar{r}} \right)^{\frac{1}{1-\alpha}} . \tag{8}$$

Government bond purchases and the sovereign's default threshold are given by $b^* = d + k - \ell$ *and* $\tau(\widehat{A}_S^*) = 0$*, respectively.*

- *Finally, for* $S > \bar{S}$ *there is no equilibrium.*

Proposition 1 shows how the relationship between bank risk-taking and sovereign default risk is shaped by the level of government debt. When this stock is low, the required tax burden on the domestic economy, if the sovereign repays, is also low. This implies a low risk of a sovereign default and therefore a low interest rate r_g.

The bank, which is subject to limited liability, still has an incentive to 'gamble' – formally captured by the conditional expectation term in Equation (7) – and holds a relatively risky portfolio. Since the bank's likelihood to default is greater than the government's, we have $\widehat{A}_S < \bar{A}$ in the asymmetric nexus. Importantly, since the government always repays in states of the world where the bank survives, the bank perceives government bonds as 'risk-free' investments.

As the stock of government debt increases, so too does the risk of sovereign default. At the same time, the likelihood that the bank fails, conditional on the government repaying, remains relatively unchanged. For a sufficiently large stock of debt, we obtain that $\bar{A} < \widehat{A}_S$, and so bank and sovereign default are perfectly synchronised around $\widehat{A}_S$ in the symmetric nexus regime. Again, since the government always repays in states of the world where the bank survives, government bonds are viewed as safe investments by the bank.

Under both the asymmetric and symmetric nexus, the bank ignores states of the world where the government defaults. The reason for this is that the bank always defaults in those states as well and is protected against further losses by limited liability. Sovereign default risks matter only indirectly through their effect on the equilibrium rate of return on bonds. This result will be important in the discussion below.

Finally, if the stock of debt is very high, then the rational expectations equilibrium does not exist. Such a situation can be interpreted as a market breakdown where the government always defaults for sure and the interest rate it is charged is infinitely large.

3.2 Comparative statics

Next, we show how the Pareto efficient equilibrium outcomes for bank's investment, sovereign default risk and bank default risk change with changes in bank capital κ, the stock of government debt S, and the refinancing cost $\bar{r}$.

Proposition 2. *Under the asymmetric nexus, increases in bank capital, k, have an ambiguous effect on the bank's investment, ℓ^*, while under the symmetric nexus, it is increasing in capital, i.e.,* $\frac{d\ell^*}{dk} > 0$.

Mechanism. Under the asymmetric nexus, the effect from an increase in bank capital can be decomposed into a direct effect on the bank's profits, subject to limited liability, and an indirect – general equilibrium – effect on the government's incentives to default. Accordingly, the direct effect of having more capital is that the bank is better able to withstand adverse shocks and retain positive equity value. But, since the bank has more 'skin in the game' it seeks to reduce the riskiness of its portfolio. To this end, the bank increases its holdings of government bonds, which the bank views as risk-free since sovereign default only occurs for realisations of the shock where the bank fails as well.

The indirect effect from having more capital is a reduction in the extent to which investment is crowded out when the bank purchases government bonds. This improves the government's incentives to repay, which reduces the return that the bank earns on government bonds. The better capitalised bank responds, in turn, by reducing its holdings of government bonds, which counteracts the direct effect leading to an ambiguous total effect.

For the symmetric nexus, by contrast, the direct effect on the bank's profits is not present since the bank adopts the government's default threshold as its own and cannot influence this via its portfolio choice. Only the indirect effect via the government's default incentives is present, implying that following an increase in its capital, the bank reduces its holdings of government bonds and increases its investments instead.

Proposition 3. *In both cases, bank investment is decreasing following an increase in either the stock of government debt, S, or the refinancing cost, $\bar{r}$.*

Mechanism. Under both the asymmetric nexus and symmetric nexus, the amount of government debt to refinance, S, does not directly impact the bank's incentives to invest or hold government bonds. Instead, the increase in S implies a higher tax burden if the government repays. This, in turn, reduces the government's incentives to repay, which leads to an increase in the interest rate, r_g^*, required by bond holders to refinance the government's debt. This indirect equilibrium effect leads to the bank rebalancing its portfolio towards holding more government bonds.

While an increase in $\bar{r}$ also induces a similar indirect effect, there is also the direct effect of increasing the opportunity cost of investing in projects. This reduces the bank's incentive from investing in favour of holding more government bonds. In sum, both the direct and indirect reinforce each other leading to a decline in investment.

Proposition 4. *The government's default threshold, $\widehat{A}_S^*$ is decreasing in bank capital, k, and increasing in the stock of debt to refinance, S, and the refinancing cost, $\bar{r}$.*

Mechanism. As bank capital increases, there is less crowding out of investment as the bank purchases government bonds. This improves the government's incentives to repay and reduces the interest rate, r_g^*. But this leads to a countervailing equilibrium effect, whereby the yield on government bonds is reduced. This weakens the bank's incentives to hold them. Thus, the increase in bank capital substitutes for the commitment effect that bank holdings of government bonds provide for the government.

The effects from an increase in either S or $\bar{r}$ can be similarly decomposed. First, an increase in either variable weakens the government's incentives to repay, which increase the interest rate, r_g^*. But, insofar that the bank reallocates its portfolio towards holding more government bonds, this will improve the government's incentives to repay, which is a countervailing effect on r_g^*.

Corollary 1. *In the symmetric nexus, bank default risk is decreasing in the bank's capital, but is increasing in the stock of debt for refinancing, S, and the refinancing cost, $\bar{r}$. The effects for the asymmetric nexus are ambiguous.*

The results for the symmetric nexus follow directly from Proposition 4, where the bank adopts the sovereign's default threshold as its own. Thus, our results on sovereign risk-premia follow through to describe bank risk-premia, and how these are driven by macro and fiscal factors.

For the asymmetric nexus, however, the comparative static exercises on the bank's default threshold are all ambiguous. As an illustration, consider the effect of an increase in bank capital on the bank's failure threshold. This can be decomposed into three effects: (i) a direct effect, (ii) an indirect effect via the bank's investment choice and (iii) an indirect effect via the sovereign's default threshold. The direct effect of an increase in bank capital is for the bank default threshold to decrease, thereby reducing the incidence of bank default.

But, at the same time, since an increase in bank capital also reduces sovereign default risk, the yield on government bonds is reduced, which reduces the net return that the bank earns. This increases the likelihood of bank default. Finally, as bank capital increases, the bank reduces its investments and favours holding more government bonds in the asymmetric nexus. This, in turn, also increases the likelihood of bank default. In sum, while the direct effect of an increase in bank capital is to reduce the likelihood of bank default, the indirect effects increase this likelihood instead.

4 When is financial repression socially optimal?

In our analysis thus far, banks failed to internalise how their purchases of government bonds influenced the government's decision to repay and the bond return. In this section, we derive the portfolio allocation chosen by a social planner who maximises expected aggregate domestic utility but still has to abide by the participation constraint of foreign investors.

By increasing banks' holdings of government bonds, the planner trades off increasing the government's incentives to repay versus the crowding-out of real investments. We subsequently show that the welfare effects of financial repression, i.e., formally requiring banks to hold more bonds than they would voluntary choose, depend on the cost of default and the type of the nexus.

4.1 Planner's problem

The planner seeks to maximise aggregate domestic utility of bankers and households subject to the government's commitment friction to repay. Our welfare benchmark is constrained efficiency, and the planner's problem is

$$\begin{aligned}
\max_{b,\ell,r_g,\widehat{A}_S} \quad & \int_0^{\widehat{A}_S} V^D(A)\mathrm{d}F(A) + \int_{\widehat{A}_S}^{\infty} V^R(A)\mathrm{d}F(A) \qquad (9)\\
& \text{subject to}\\
& \ell + b = d + k\\
& 1+\bar{r} - \big(1+r_g\big)\big(1-F(\widehat{A}_S)\big) = 0\\
& \frac{\big(S-b\big)\big(1+r_g\big)}{\delta \ell^{\alpha}} - \widehat{A}_S = 0
\end{aligned}$$

where $V^R(A)$ and $V^D(A)$ are aggregate domestic utility if the government repays and defaults and are defined by Equation (3) and Equation (4), respectively. The optimisation is subject to three constraints. The first is the balance sheet identity for banks. The second is the participation constraint for foreign investors, from which we determine the price of government bonds. The third constraint defines the government default threshold as a function of banks' portfolio choices.

Proposition 5. *The planner's choice for the optimal level of investment is given by*

$$\begin{aligned}
&\frac{\alpha(\ell^{SP})^{\alpha-1}}{1-F(\widehat{A}_S^{SP})}\left[\big(1-\delta\big)\int_0^{\widehat{A}_S^{SP}} A\,dF(A) + \int_{\widehat{A}_S^{SP}}^{\infty} A\,dF(A)\right]\\
&-\Big[S-\big(d+k-\ell^{SP}\big)\Big] \times \left.\frac{\partial r_g^*}{\partial \ell}\right|_{\ell^{SP},\widehat{A}_S^{SP}} = \frac{1+\bar{r}}{1-F(\widehat{A}_S^{SP})}, \qquad (10)
\end{aligned}$$

where the sovereign default threshold is given by

$$\widehat{A}_S^{SP} = \frac{\big(S\;-\;\big(d+k-\ell^{SP}\big)\big)}{\delta\,(\ell^{SP})^{\alpha}}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S^{SP})}\right),$$

and the interest rate on government bonds, r_g^*, is derived from the foreign investors' binding participation constraint.

Compared with the allocation chosen by the representative bank, we note two striking differences. First, the planner also cares about aggregate domestic utility in states of the world where the government defaults. The bank, in contrast, ignores outcomes in these states. This is because the bank is protected by limited liability and the government only defaults in states where the bank also defaults. And second, the planner accounts for how changes in the bank's investment influences the interest rate charged on government bonds, and thereby the tax revenue transferred to foreign investors.

Proposition 6. *There exist two bounds for the cost of sovereign default on the bank's investment, $\bar{\delta}$ and $\underline{\delta}$, where $\bar{\delta} > \underline{\delta}$ such that:*

	Asymmetric Nexus ($S \leq \underline{S}$)	*Symmetric Nexus* ($\underline{S} < S \leq \bar{S}$)
$\delta < \underline{\delta}$	$\ell^{SP} > \ell^*$	$\ell^{SP} > \ell^*$
$\delta \in (\underline{\delta}, \bar{\delta})$	$\ell^{SP} < \ell^*$	$\ell^{SP} > \ell^*$
$\delta > \bar{\delta}$	$\ell^{SP} < \ell^*$	$\ell^{SP} < \ell^*$

The optimality of financial repression depends on the economic losses resulting from a sovereign default and the type of the nexus. In general, the benefit from banks holding more government bonds is to improve the incentives of the government to repay, which reduces the interest rate on government bonds and the tax burden on the domestic economy, insofar that the government chooses to repay. The cost from the bank holding more government bonds is the crowding-out of domestic investment and therefore output in $t = 1$.

Proposition 6 shows that if the real cost of a sovereign default is large, $\delta > \bar{\delta}$, the bank holds too few government bonds in the competitive equilibrium, relative to the planner's allocation. Since the bank does not internalize directly the relatively high cost of o default, it invests too much in the real economy. This exacerbates the potential costs from a default. At time same time, however, the costs borne by the government from repaying are higher since more of its debt is held by foreign investors. By forcing the bank to hold more government bonds, the planner continues to maintain incentives for the government to repay, while reducing the net costs from doing so.

If, however, the real cost of a sovereign default is low, $\delta < \underline{\delta}$, avoiding default becomes relatively less important. In the competitive equilibrium banks hold too many government bonds. They do not internalise that their investment choice crowds-out too much real investment which in turn leads to a lower tax base in the next period. The planner, in contrast, chooses an allocation where banks hold less government bonds than in the competitive equilibrium.

For intermediate values, $\delta \in [\underline{\delta}, \bar{\delta}]$, the social planner engages in financial repression only in the asymmetric nexus. In this regime, limited liability for banks plays a role in shaping their risk-taking. In particular, banks reduce their holdings of safe government bonds and increase their level of investment, which is risky. But such risk-taking by banks leads to foreign investors holding too much sovereign debt, which weakens the government's incentives to repay. To remedy this, the planner requires banks to reduce investments and hold more government bonds.

Our analysis does not directly address how such financial repression, which in our case, could also mean to force banks to hold less bonds, may be implemented in practice. However, there are several tools, some already existing, that could be used. One would be to use the tax system to either tax bonds more or less than real investment projects. Another option would be to impose explicit limits and restrictions on banks purchases of government bonds if the government wanted to reduce banks' holdings of its bonds. If it wanted to increase it, it could, for example, increase liquidity requirements, which typically require banks to hold more domestic sovereign debt.

5 Welfare effects of financial repression and implications for recent policy proposals

Our normative results suggest that financial repression can be socially optimal. Forcing banks to increase their holdings of domestic government bonds is particularly valuable when the costs of a sovereign default on the domestic economy are high and the resulting costs in terms of crowding-out real investments are low. The key effect of such an intervention is that it increases the government's incentives to repay, which in turn reduces the price it must pay to refinance its debts.

This has important consequences for assessing the proposals which have been introduced to curb banks' holdings of domestic government bonds in the aftermath of the crisis. One such proposal, for example, envisions introducing an upper bound on the ratio between a bank's holdings of domestic sovereign debt and the bank's capital (European Systemic Risk Board, 2015). While such a 'large exposure limit' already exists for other bank assets, sovereign exposures are currently exempt under the Basel III regulation. A related proposal suggests introducing risk-weights for banks' sovereign debt exposures in calculating capital requirements (Basel Committee on Banking Supervision, 2017). Finally, a recent market-based approach proposal suggests establishing special financial vehicles to buy up sovereign debt from euro area banks to be used for securitisation (European Commission, 2018).

The results in Proposition 6 allow us to qualitatively assess the efficacy of such regulations. Suppose, for example, sovereign default is disorderly and results in large losses in the real economy, i.e., $\delta > \bar{\delta}$.[9] Then, irrespective of the nexus, reducing banks' holdings of domestic sovereign debt increases the government's incentives to default. This results in a higher interest rate being charged on government bonds and is overall detrimental to the domestic economy.

If, on the other hand, the default proceeds in an orderly manner, for example, facilitated by a sovereign debt restructuring mechanism (e.g., Krueger, 2002; Brookings Committee on International Economic Policy and Reform (CIEPR), 2013 and Deutsche Bundesbank, 2016), then the costs to the real economy from the default are muted, i.e., $\delta < \underline{\delta}$. In this case, and again irrespective of the nexus, social welfare is improved when banks lower their holdings of government bonds and invest more in the real economy.

Finally, if the cost of default is in an intermediate range, $\delta \in [\underline{\delta}, \bar{\delta}]$, the welfare effects of financial repression depend crucially on the nexus. In the asymmetric nexus, which occurs when the debt level is low, financial repression can improve welfare. While, in the symmetric nexus, which occurs with the debt level is high, the opposite

[9] Hebert and Schreger (2017) estimate that between January 2011 and July 2014, when Argentina defaulted on bond holders who had previously accepted to restructure their debt, the value of Argentine firms reduced by about 30%.

is true. Our result is, thus, distinct from that of Reinhart and Sbrancia (2015) and Chari et al. (2020), who argue that financial repression is beneficial only in situations with exceptionally high debt levels.

Our model, thus, suggests that the design of policy to regulate banks' holdings of domestic government bonds must take into account the cost of a sovereign default and the type of the nexus between sovereign risk and banking risk-taking, which in turn depends crucially on the level of debt.

6 Conclusion

We have developed a model of bank risk-taking with strategic sovereign default risk. Domestic banks can either invest in real projects or purchase government bonds. While an increase in purchases of government bonds crowds out profitable investment, it nevertheless improves the government's incentives to repay and therefore reduces the bond price. We document three key results.

First, the connection between bank risk-taking and sovereign default risk depends crucially on the level of government debt. An asymmetric nexus in which banks always default when the sovereign defaults, but not vice versa, arises for low levels of debt. While, when debt levels are high, we obtain a symmetric nexus where bank and sovereign default are perfectly synchronised.

Second, banks' equity values are discontinuous with respect to aggregate shock in the symmetric nexus. In this case, the banks' default thresholds are given by the sovereign's default threshold and therefore exogenous to each individual bank. Portfolio adjustments of a bank will not affect its survival probability. In the asymmetric nexus, however, banks' optimal portfolio decision influence their default thresholds.

Third, we show that banks can hold too much or too few government bonds in the competitive equilibrium. If default costs are high, or the economy is in a symmetric nexus, banks under-invest in government bonds. In such situations, regulations aimed at limiting banks' holdings of sovereign debt are welfare reducing.

We also show that our model results are in line with recent empirical evidence on the sovereign debt crisis in the euro zone. The observed increase in banks' holdings of domestic sovereign debt can be a market outcome in our model. It does not require moral suasion. However, and more importantly, our normative results show that if there was moral suasion, it might have been welfare improving.

There are, at least, two important directions for future research. First, the output loss in our model occurs when the government defaults and not when banks default. In the symmetric nexus, bank default and government default are synchronised, so we may attribute the cost to a systemic banking crisis. In the asymmetric case, however, there are situations when only banks default. While introducing a cost of bank default into the government's problem complicates the analysis, it would yield additional insights that are relevant outside crises periods. Second, it would be interesting to extend our model to a dynamic setting in order to be able to quantitatively assess the mechanism.

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A Proof of Proposition 1

Let $\pi^A = \int_{\bar{A}(\ell)}^{\infty} \big(A\,\ell^{\alpha} + (1+r_g)\big(d + k - \ell\big) - d\big)\, dF(A)$ denote the bank's objective function under the asymmetric nexus, where $\bar{A}(\ell) = \frac{d-(1+r_g)(d+k-\ell)}{\ell^{\alpha}}$ is the bank's default threshold. The objective function under the symmetric nexus is $\pi^S = \int_{\widehat{A}_S}^{\infty} \big(A\,\ell^{\alpha} + (1+r_g)\big(d + k - \ell\big) - d\big)\, dF(A)$, where $\widehat{A}_S$ is the government's default threshold.

To determine the optimal levels of investment under the different nexus, we first take the derivatives of the objective functions with respect to ℓ. This yields

$$
\begin{aligned}
\pi_{\ell}^{A} &= \alpha\,\ell^{\alpha-1}\int_{\bar{A}(\ell)}^{\infty} A\,dF(A) - (1+r_g)\Big(1 - F\big(\bar{A}(\ell)\big)\Big)\,, \\
\pi_{\ell}^{S} &= \alpha\,\ell^{\alpha-1}\int_{\widehat{A}_S}^{\infty} A\,dF(A) - (1+r_g)\big(1 - F(\widehat{A}_S)\big)\,.
\end{aligned}
$$

Optimal investment under the different nexus regimes are given by the first-order conditions, $\pi_{\ell}^{A}(\ell^*) = 0$ and $\pi_{\ell}^{S}(\ell^*) = 0$. Under the symmetric nexus, an explicit solution for ℓ^* is obtained, which is unique. For the asymmetric nexus, under the condition that $1 + r_g < \frac{\alpha(d+k)^{\alpha-1}}{1-F(d/(d+k)^{\alpha})} \int_{d/(d+k)^{\alpha}}^{\infty} A dF(A)$, we can appeal to the intermediate value theorem for ℓ^* to be unique.

Next, since domestic banks are price takers, the price of sovereign bonds are determined by foreign investors according to Equation (6), which on substituting into the first-order conditions yields our results for optimal investment.

To derive the critical sovereign default threshold, we rewrite Equation (5) as

$$
\widehat{A}_S = \frac{S - (d + k - \ell)}{\delta\,\ell^{\alpha}}\big(1 + r_g\big)\,.
$$

Substituting out $1 + r_g$ using Equation (6) yields our result that the equilibrium sovereign default threshold is implicitly defined by $\tau(\widehat{A}_S^*) = 0$, where

$$\tau(\widehat{A}_S) \equiv \widehat{A}_S - \frac{S - (d + k - \ell)}{\delta\,\ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right). \tag{11}$$

Market failure. The function $\tau(\widehat{A}_S)$ is globally concave. We derive this by noting that

$$\tau''(\widehat{A}_S) = -\frac{\lambda^2\big(S-(d+k-\ell)\big)}{\delta\ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right),$$

which is strictly negative as long as $d + k - \ell < S$, i.e., the domestic bank does not hold all government bonds. This is always true since, at the margin, foreign investors must hold some government bonds to determine the price. We also note that $\lim_{\widehat{A}_S\to 0}\tau(\widehat{A}_S) < 0$ and $\lim_{\widehat{A}_S\to T}\tau(\widehat{A}_S) = -\infty < 0$. This implies that if $\tau(\widehat{A}_S)$ crosses the x-axis, then it does so twice, implying two distinct equilibria. But, it is also possible that $\tau(\widehat{A}_S)$ does not cross the x-axis, and hence there is market failure and no equilibrium solution. The market failure condition is derived as the point, $\widehat{A}^{MF}$ where the curve $\frac{\lambda^2\left(S-(d+k-\ell)\right)}{\delta\ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}^{MF})}\right)$ is tangential to the 45-degree line, i.e., $\tau'(\widehat{A}^{MF}) = 1$. We obtain that

$$\widehat{A}^{MF} = F^{-1}\left(1 - \frac{\lambda\big(S-(d+k-\ell)\big)}{\delta\ell^\alpha}\big(1+\bar{r}\big)\right),$$

where F^{-1} is the inverse cumulative distribution function for the TFP shock. As long as $\tau(\widehat{A}^{MF}) \geq 0$, there is no market failure, where

$$\tau(\widehat{A}^{MF}) = \widehat{A}^{MF} - \frac{S-(d+k-\ell)}{\delta\ell^\alpha}\left(\frac{1+\bar{r}}{\frac{\lambda\left(S-(d+k-\ell)\right)\left(1+\bar{r}\right)}{\delta\ell^\alpha}}\right) = \widehat{A}^{MF} - \frac{1}{\lambda}.$$

Rearranging the condition, we obtain that as long as $S \leq S^{MF}$, there is no market failure, where $\bar{S}$ is implicitly given by

$$F^{-1}\left(1 - \frac{\lambda\big(\bar{S} - (d + k - \ell)\big)}{\delta \ell^{\alpha}}\big(1 + \bar{r}\big)\right) - \frac{1}{\lambda} = 0\,.$$

Bound for asymmetric nexus. For the asymmetric nexus, we require $\widehat{A}_S < \bar{A} < \tilde{A}$. In the vicinity of the Pareto efficient equilibrium, $\tau_{\widehat{A}_S} > 0$. This implies that to be in the asymmetric nexus, we must have that $\tau(\bar{A}) > 0$. We can express the equilibrium condition as follows.

$$S < \frac{\delta d}{1 + \bar{r}}\big(1 - F(\bar{A}(\ell^*))\big) + \big(d + k - \ell^*\big)\left[1 - \frac{\delta\big(1 - F(\bar{A}(\ell^*))\big)}{1 - F(\widehat{A}^*_S)}\right] \equiv \underline{S}\,.$$

Interval for symmetric nexus. In general, it is also possible to obtain the ordering of thresholds whereby $\bar{A} < \tilde{A} < \widehat{A}_S$. This occurs whenever $\tau(\tilde{A}) < 0$, and can be expressed as

$$S > \frac{\delta d}{1 + \bar{r}}\left(\frac{1 - F(\tilde{A}(\ell^*))}{1 - \delta}\right) + (d + k - \ell^*) \equiv \tilde{S}\,.$$

However, if $\bar{S} < \tilde{S}$, then the market equilibrium breaks down before we reach the new regime. This requires $\tau(\widehat{A}^{MF}) < \tau(\tilde{A})$, which on rearranging yields

$$\lambda > \widehat{\lambda} \equiv \left[\widehat{A}^{MF} - \tilde{A}^* + \frac{S - (d + k - \ell^*)}{\delta \ell^{\alpha}}\left(\frac{1 + \bar{r}}{1 - F(\tilde{A}^*)}\right)\right]^{-1}. \qquad (12)$$

B Proof of Propositions 2 - 4 and Corollary 1

In this section we investigate how changes to the lending rate for foreign investors, $\bar{r}$, banker's endowment, k, and stock of debt to refinance for the sovereign, S, influence the equilibrium level of investment. In general, we can decompose the effects into direct effects via the bank's first-order condition, and an indirect effect via the pricing of government bonds. Since the pricing of government bonds is the same under both the asymmetric nexus and symmetric nexus, we first describe the partial effects of changes in the exogenous variables on $\widehat{A}_S^*$. We obtain the following.

$$
\begin{aligned}
\tau_{\widehat{A}_S}(\widehat{A}_S^*) &= 1 - \lambda \widehat{A}_S^* > 0 \\
\tau_\ell &= \frac{1}{\delta \ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right)\left[-1 + \frac{\alpha \ell^{\alpha-1}}{\ell^\alpha}\left(S - (d+k-\ell)\right)\right] < 0 \\
\tau_{\bar{r}} &= -\frac{S-(d+k-\ell)}{\delta \ell^\alpha \big(1-F(\widehat{A}_S)\big)} < 0 \\
\tau_k &= \frac{S}{\delta \ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right) > 0 \\
\tau_S &= -\frac{1}{\delta N \ell^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right) < 0\,.
\end{aligned}
$$

We now turn to the two nexus and first determine the partial effects of changes in the exogenous parameters on the bank's optimal choice and subsequently derive the total effects using Cramer's rule.

Asymmetric Nexus

First, we show that the optimal level of investment is a maximum. This is given by showing $\pi^A_{\ell\ell}(\ell^*) < 0$. We obtain that

$$\begin{aligned}
\pi^A_{\ell\ell} &= \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\bar{A}(\ell)}^{\infty} A\, dF(A) - \alpha\ell^{\alpha-1}\bar{A}(\ell)f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \ell} + \frac{1+\bar{r}}{1-F(\widehat{A}_S)}f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \ell} \\
&= \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\bar{A}(\ell)}^{\infty} A\, dF(A) - f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \ell}\left\{\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\},
\end{aligned}$$

where $\frac{\partial \bar{A}}{\partial \ell} = -\frac{1}{\ell^\alpha}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right]$. At the equilibrium, ℓ^*, we get

$$\begin{aligned}
\pi^A_{\ell\ell}(\ell^*) &= \frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\big(1+\bar{r}\big)\frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} + \frac{f(\bar{A}(\ell^*))}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2 \\
&= \big(1-F(\bar{A}(\ell^*))\big)\left[\frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\frac{1+\bar{r}}{1-F(\widehat{A}_S)} + \frac{\lambda}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2\right].
\end{aligned}$$

Since the first term in the square brackets is negative, while the second is positive, if the hazard rate satisfies, $\lambda < \bar{\lambda}$, then $\pi^A_{\ell\ell}(\ell^*) < 0$, where the upper bound is given by the solution to

$$\frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\frac{1+\bar{r}}{1-F(\widehat{A}_S)} + \frac{\bar{\lambda}}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2 = 0\,.$$

Next, we derive the partial effects from increases in the sovereign default threshold, $\widehat{A}_S$, risk-free rate, $\bar{r}$, stock of debt, S, and bank capital, k, on the optimal investment.

We obtain that

$$
\begin{aligned}
\pi^A_{\ell\widehat{A}_S} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \widehat{A}_S}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right] - \lambda(1+\bar{r})\frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} \\
\pi^A_{\ell\bar{r}} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \bar{r}}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right] - \frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} \\
\pi^A_{\ell S} &= 0 \\
\pi^A_{\ell k} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial k}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right]
\end{aligned}
$$

Clearly, the signs for $\pi^A_{\ell\widehat{A}_S}$, $\pi^A_{\ell\bar{r}}$ and $\pi^A_{\ell k}$ depend on the sign of $\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}$, which at the optimum ℓ^* can we re-written as $\alpha(\ell^*)^{\alpha-1}\big[\bar{A}(\ell^*) - \frac{\int_{\bar{A}(\ell^*)}^{\infty} A dF(A)}{1-F(\bar{A}(\ell^*))}\big] < 0$. Hence, $\pi^A_{\ell\widehat{A}_S} < 0$, $\pi^A_{\ell\bar{r}} < 0$ and $\pi^A_{\ell k} < 0$.

The determinant of the Jacobian matrix is

$$
|J^A| = \begin{vmatrix} \pi^A_{\ell\ell} & \pi^A_{\ell\widehat{A}_S} \\ \tau_\ell & \tau_{\widehat{A}_S} \end{vmatrix} < 0 \,.
$$

The comparative statics for the optimal level of investment are, thus, as follows.

$$
\begin{aligned}
\frac{d\ell^*}{d\bar{r}} &= \frac{\begin{vmatrix} -\pi^A_{\ell\bar{r}} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_{\bar{r}} & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} < 0 \,, \quad \frac{d\ell^*}{dk} = \frac{\begin{vmatrix} -\pi^A_{\ell k} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} \,, \\
\frac{d\ell^*}{dS} &= \frac{\begin{vmatrix} -\pi^A_{\ell S} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_S & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} < 0 \,, \quad \frac{d\ell^*}{d\delta} = \frac{\begin{vmatrix} -\pi^A_{\ell\delta} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_\delta & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} > 0 \,.
\end{aligned}
$$

In general the effect of a change in bank capital on investment has an ambiguous sign.

Note, however, that

$$\omega(S) \equiv \begin{vmatrix} -\pi^A_{\ell k} & \pi^A_{\ell \widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix} = -\pi^A_{\ell k}(1-\lambda \widehat{A}^*_S) + \frac{S}{\delta(\ell^*)^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)\pi^A_{\ell \widehat{A}_S}$$

is decreasing is S and at $S=0$ it is strictly positive. Thus If $\omega(\underline{S}) > 0$, then this establishes that under the asymmetric nexus, an increase in bank capital leads to a decrease in investment, i.e., $\frac{\partial \ell^*}{\partial k} < 0$. This is equivalent to requiring that

$$d > \underline{d} \equiv \frac{\pi^A_{\ell k}\left(1-\lambda \widehat{A}^*_S\right) - \frac{k-\ell^*}{\delta(\ell^*)^\alpha}\left[1-\delta\xi^*\right]\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)}{\pi^A_{\ell \widehat{A}_S}\left[\xi^* + \frac{1}{\delta(\ell^*)^\alpha}\left(1-\delta\xi^*\right)\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)\right]},$$

where $\xi^* = \frac{1-F(\bar{A}^*)}{1-F(\widehat{A}^*_S)}$. For sufficiently small k, this condition is satisfied for all d.

The total effects on the sovereign's default threshold are

$$\frac{d\widehat{A}^*_S}{d\bar{r}} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell\bar{r}} \\ \tau_\ell & -\tau_{\bar{r}} \end{vmatrix}}{|J^A|} \lesseqgtr 0\,, \quad \frac{d\widehat{A}^*_S}{dk} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell k} \\ \tau_\ell & -\tau_k \end{vmatrix}}{|J^A|} < 0$$

$$\frac{d\widehat{A}^*_S}{dS} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell S} \\ \tau_\ell & -\tau_S \end{vmatrix}}{|J^A|} > 0\,, \quad \frac{d\widehat{A}^*_S}{d\delta} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell\delta} \\ \tau_\ell & -\tau_\delta \end{vmatrix}}{|J^A|} < 0\,.$$

Symmetric Nexus

As before, we first show that the optimal level is a maximum, which requires $\pi^S_{\ell\ell}(\ell^*) < 0$. We readily obtain

$$\pi^S_{\ell\ell} = \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\widehat{A}_S}^{\infty} A\,dF(A) < 0\,.$$

Next, for the partial effects of a change in $\widehat{A}_S$, $\bar{r}$, k and S, we obtain $\pi^S_{\ell\widehat{A}_S} = -\alpha\ell^{\alpha-1}\,\widehat{A}_S f(\widehat{A}_S) < 0$, $\pi^S_{\ell\bar{r}} = -1 < 0$, $\pi^S_{\ell k} = 0$, $\pi^S_{\ell S} = 0$, and $\pi^S_{\ell\delta} = 0$.

The determinant of the Jacobian matrix is

$$|J^S| = \begin{vmatrix} \pi^S_{\ell\ell} & \pi^S_{\ell\widehat{A}_S} \\ \tau_\ell & \tau_{\widehat{A}_S} \end{vmatrix} < 0\,.$$

The comparative statics for the optimal level of investment are, thus, as follows.

$$\begin{aligned}
\frac{d\ell^*}{d\bar{r}} &= \frac{\begin{vmatrix} -\pi^S_{\ell\bar{r}} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_{\bar{r}} & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} < 0\,, \quad \frac{d\ell^*}{dk} = \frac{\begin{vmatrix} -\pi^S_{\ell k} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} > 0 \\
\frac{d\ell^*}{dS} &= \frac{\begin{vmatrix} -\pi^S_{\ell S} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_S & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} < 0\,, \quad \frac{d\ell^*}{d\delta} = \frac{\begin{vmatrix} -\pi^S_{\ell\delta} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_\delta & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} > 0\,.
\end{aligned}$$

The total effects on the sovereign's default threshold are

$$
\begin{aligned}
\frac{d\widehat{A}_S^*}{d\bar{r}} &= \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell\bar{r}} \\ \tau_\ell & -\tau_{\bar{r}} \end{vmatrix}}{|J^S|} \lesseqgtr 0\,, \quad \frac{d\widehat{A}_S^*}{dk} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell k} \\ \tau_\ell & -\tau_k \end{vmatrix}}{|J^S|} < 0 \\
\frac{d\widehat{A}_S^*}{dS} &= \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell S} \\ \tau_\ell & -\tau_S \end{vmatrix}}{|J^S|} > 0\,, \quad \frac{d\widehat{A}_S^*}{d\delta} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell\delta} \\ \tau_\ell & -\tau_\delta \end{vmatrix}}{|J^S|} < 0\,.
\end{aligned}
$$

Finally, since the bank default threshold is identical to the sovereign default threshold, the comparative statics are identical.

C Proof of Propositions 5 - 6

We can re-write the planner's problem as $\max_\ell W(\ell)$, where

$$
\begin{aligned}
W(\ell) \;&\equiv\; \ell^\alpha \left[(1-\delta) \int_0^{\widehat{A}_S(\ell)} A\, dF(A) + \int_{\widehat{A}_S(\ell)}^{\infty} A\, dF(A) \right] \\
&- \; \Big(1 + r_g^*(\ell)\Big)\Big(S - (d + k - \ell)\Big)\,\Big(1 - F(\widehat{A}_S(\ell))\Big)\;,
\end{aligned}
$$

where $r_g^*(\ell)$ is derived from the foreign investors' binding participation constraints such that $\frac{\partial r_g^*}{\partial \ell} > 0$. The result in Equation (10) following immediately from the first-order condition, $W_\ell(\ell^{SP}) = 0$, where all partial effects via the sovereign default threshold cancel out. We also assume that this optimum is a maximiser, i.e., $W_{\ell\ell}(\ell^{SP}) < 0$.

We next compare the level of investment from the competitive equilibrium, ℓ^*, versus the social planner's allocation, ℓ^{SP}. To this end, if there is too much investment in the real economy under the competitive solution, i.e., $\ell^* > \ell^{SP}$, then this would imply that $W_\ell(\ell^*) < 0$. We consider the competitive equilibrium investment under the asymmetric nexus and symmetric nexus in turn.

Asymmetric nexus. Evaluating the planner's first-order condition at the competitive equilibrium, we get

$$\begin{aligned} W_\ell(\ell^*) &= \frac{\alpha\,(\ell^*)^{\alpha-1}}{1-F(\widehat{A}_S^*)}(1-\delta)\int_0^{\widehat{A}_S^*} A\,dF(A) - \Big(S-(d+k-\ell^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\bigg|_{\ell=\ell^*} \\ &- \frac{\alpha\,(\ell^*)^{\alpha-1}}{1-F(\bar{A}^*)}\int_0^{\bar{A}^*} A\,dF(A)\,. \end{aligned}$$

Denoting by $\Omega \equiv \int_0^{\widehat{A}_S^*} A\,dF(A) - \frac{1-F(\widehat{A}_S^*)}{1-F(\bar{A}^*)}\int_0^{\bar{A}^*} A\,dF(A) < 0$, we have that the level of investment under the competitive equilibrium is too high whenever

$$\delta > \underline{\delta} \equiv 1 - \frac{\Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\Big|_{\ell=\ell^*}}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)} + \frac{\Omega}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)}$$

Symmetric nexus. In this case, we have that

$$W_\ell(\ell^*) = \alpha\,(\ell^*)^{\alpha-1}\,(1-\delta)\int_0^{\widehat{A}_S^*} A\,dF(A) - \Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\bigg|_{\ell=\ell^*}\,.$$

Thus, the level of investment under the competitive equilibrium is too high whenever

$$\delta > \bar{\delta} \equiv 1 - \frac{\Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\Big|_{\ell=\ell^*}}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)}$$

Finally, since $\Omega < 0$, it follows that $\underline{\delta} < \bar{\delta}$.